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Information Entropy and the Variability of Space–Time Movement Error

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ABSTRACT. The authors investigated the effects of movement time and movement distance on the information entropy and variability of spatial and temporal error in a discrete aiming movement. In Experiment 1, the authors held movement distance (100 mm) constant and manipulated 11 movement times (300–800 ms) of 8 participants. In Experiment 2, the authors tested 6 movement distances at 2 given movement times (15–60 mm at 300 ms; 40–240 mm at 800 ms) in 8 participants. The variability and entropy for spatial error increased with average movement velocity, whereas the variability and entropy for temporal error decreased as a function of average movement velocity. The common variance between variable error and entropy averaged about 84% and 72% for spatial and temporal errors, respectively, suggesting that the probabilistic approach of entropy reveals features that are not present in the standard deviation index of variability. The findings provide further evidence that information entropy may be a useful single-index representation of variability in the movement speed–accuracy relation.

Key words: information entropy, movement speed and accuracy, variability

There have been many efforts to describe and understand the relation between movement speed and accuracy. Several relations between movement speed and the variability of movement outcome have been descriptively identified, including linear (Schmidt, Zelaznik, Hawkins, Frank, & Quinn, 1979), logarithmic (Fitts, 1954), square root (Meyer, Abrams, Kornblum, Wright, & Smith, 1988), and logistic (Hancock & Newell, 1985) functions. Those different descriptions of the movement speed–accuracy relation have been linked to different theoretical perspectives, including information processing (Fitts), impulse variability (Schmidt et al.), and optimized stochastic control (Meyer et al.).

Here, we used a probabilistic account of the uncertainty of movement error, that is, information entropy (Gatlin, 1972; Shannon, 1948; Shannon & Weaver, 1949) to examine the traditional distributional account of the variability of spatial and temporal errors. Investigators have relied on normal distributional indexes of variability to describe the relationship between the functions for movement speed and accuracy. The most common index of movement variability is variable error (*VE*), which is calculated as the standard deviation of the spatial or temporal error. The standard deviation measure is based on the properties of a distribution, and in a normal distribution it has a special role in capturing the average deviations from the mean. Investigators also use the coefficient of variation (*CV*, i.e., standard deviation divided by mean) to provide a relative measure of variability to the spatiotemporal properties of the movement; *CV* is useful for contrasting variability over different task conditions. In the Fitts's (1954) law protocol, the measure of variation is in essence the range of the distribution as reflected in the target width.

There are several limitations to the use of distributional measures of outcome variability (Hancock & Newell, 1985; Newell & Hancock, 1984). A significant problem is that the assumption of a normal distribution does not hold in most conditions of the speed–accuracy relation. There are departures from the properties of a normal distribution over the potential speed–accuracy window, and those departures from a Gaussian function are not a reflection of sampling problems. Rather, there are systematic changes in skewness and kurtosis as a function of the spatiotemporal demands of the task (Hancock & Newell; Kim, Carlton, Liu, & Newell,

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1999). The departures from a normal distribution are significant because it has been found that changes in the higher order third and fourth moments that determine skewness and kurtosis, respectively, can, in and of themselves, change the estimate of the standard deviation even when the range of the scores in the distribution remains unchanged (Newell & Hancock).

Lai, Mayer-Kress, Sosnoff, and Newell (2005) examined the variability of movement outcome in relation to the direct measure of the probabilities of the movement outcome. That approach is built directly on the concept of information entropy (Cover & Thomas, 1991; Gatlin, 1972; Shannon & Weaver, 1949); that is, probabilities are the foundation of the measure of entropy. Using the measure of information entropy, Lai et al. showed that the distribution over a series of trials at points of the movement trajectory and movement outcome are not normal. Their study provided preliminary evidence that the entropy estimates of movement outcome as a function of movement speed and accuracy conditions show a different property than that produced by conventional distributional estimates such as standard deviation and *CV*. It should be noted that although Fitts's (1954) theoretical approach to the movement speed–accuracy problem was developed from information theory, he did not calculate information entropy, relying instead on a range estimate of variability to determine information capacity in the now well-known Fitts's law.

We report here two experiments in which we examined the relationship between the distributional and information entropy estimates of the speed–accuracy function over a wide range of spatial and temporal movement conditions. We were particularly interested in whether the more precise estimates of uncertainty based on the probabilities of movement outcome through information entropy produce the same or a different function of the movement speed–accuracy relation than do the distributional approaches to variability, variable error (*VE*) and *CV*. Given that the movement error data do not approximate a normal distribution over most of the movement parameter range, it follows that the inherent skewness and kurtosis may bias the variance-based approaches of *VE* and *CV* (Hancock & Newell, 1985). A problem with the distributional approaches to the assessment of movement error is how to represent the degree of skewness and kurtosis in estimates of *VE* and *CV*.

The information entropy measure has the advantage of being a single index of variability, and we sought to contrast it against the functions of dependent variables derived from the first four moments of the distribution (constant error [*CE*], *VE*, skewness, and kurtosis). We contrasted both individual- and group-average speed–accuracy relations to also determine whether averaging data in that context biases the estimate of the change in variability in the same way as has been shown for performance scores in the study of learning curves (cf. Newell, Liu, & Mayer-Kress, 2001). Our general expectation was that the resultant pattern of findings over a wider range of spatial and temporal task constraints would provide an indication of the potential usefulness of an information entropy approach to movement variability. One clear theoretical advantage of an entropy approach is that the data from different dimensions in different movement experiments can be contrasted directly because they are in the same information theoretic terms.

In Experiment 1, we contrasted the two approaches in discrete aiming tasks that had both spatial and temporal criteria over a range of movement speeds and times for a given movement amplitude. The first experiment also provided the opportunity to further test the hypothesis of a relationship between spatial and temporal errors as a function of movement speed (Hancock & Newell, 1985; Kim et al., 1999), but in an aiming task in which the limb motion had to be terminated at a target. In Experiment 2, we manipulated the spatial constraints (movement amplitude) for two given movement times. We chose one relatively long movement time (800 ms) so that the visual-feedback control would clearly influence the variability of movement outcome (Elliott, Carson, Goodman, & Chua, 1991; Woodworth, 1899). The other movement time we chose was relatively short (300 ms) so that we could test the distributional and information entropy estimates of spatiotemporal movement outcomes with minimal influence of online visual information feedback (Beggs & Howarth, 1970; Keele & Posner, 1968).

EXPERIMENT 1

Our primary goal in this experiment was to examine whether an information entropy analysis provides a different function for movement speed and accuracy than that produced by the use of the standard deviation (*VE*). Our second goal was to test the hypothesis proposed by Hancock and Newell (1985) that relative spatial and temporal variability (*CV*) in space and time has a similar logistic function when a range of movement times is manipulated over a given movement distance. We also examined the complementarity between spatial and temporal errors through both distributional and entropy approaches to the analysis of movement variability.

Method

Participants

Eight right-handed, healthy adults (6 women and 2 men) took part in the current experiment. All participants were students at The Pennsylvania State University ranging in age from 24–31 years (M = 28.13 years, SD = 2.17 years). All participants gave informed consent to the experimental procedures, which were approved for compliance with the policy of The Pennsylvania State University Institutional Review Board. Participants were given a small monetary reward for their participation.

Apparatus

The apparatus used in the current study included the following: (a) a 12- × 18-in. WACOM Intuos (Model GD-

1218-R) graphics tablet; the sampling frequency was 200 Hz, and the resolutions were 50 tablet units per mm in the *x*-axis and 48 tablet units per mm in the *y*-axis; (b) a 13-g, 151- × 12.1-mm Intuos pen (Model GP-300E); (c) a 17-in. video monitor (335 mm × 248 mm); and (d) an Audioworks computer speaker system (Model AE-48). We used Matlab Version 6.0 (Mathworks, Inc., Natick, MA) for data preparation and SPSS for Windows Version 12.0 (SPSS Inc., Chicago, IL) for statistical analysis. We used SigmaPlot 2000 for the examination of function fitting of spatial and temporal *VE* and *CV*. We set the pixels on the monitor at 800 × 600. The ratio of the distance moved on the screen and the distance moved on the board was 1:1.

Procedure

The participant sat comfortably on a chair, holding the Intuos pen and facing a computer screen that was at a distance of approximately 60 cm. The initial start position (2 mm in width) on the left and the end target (1.8 mm in width) on the right were fixed (both shown as red points on the white screen). A black, small point (1.8 mm in width) corresponding to a pen tip was displayed on the screen. To initiate a trial, we asked the participant to move the pen in a left-to-right horizontal direction over a target distance in a target time. We calculated the beginning of movement as the point at which the pen's velocity was greater than or equal to 3 mm/s for more than 30 ms. We defined the end of movement (space and time) as the initial point at which the pen's velocity was less than 3 mm/s for more than 40 ms.

An auditory tone was given once the participant held the pen within the start position for 1 s. The participant was instructed not to respond to the auditory tone as fast as possible (it was not a reaction time experiment) but to begin a trial comfortably after hearing the tone. We displayed the trajectory of the pen on the screen during each trial to provide online kinematic feedback. The movement time was presented to the participant after the completion of each trial. Participants were required to perform the discrete aiming movement in different temporal constraint (movement time) conditions but under the same spatial constraint.

Experimental Design

Movement distance in this experiment was held constant (100 mm); but the movement time ranged from 300 ms to 2,050 ms in equal increasing 175-ms increments: 300, 475, 650, 825, 1,000, 1,175, 1,350, 1,525, 17,00, 1,875, and 2,050 ms. There were 200 trials performed in each movement time condition in a blocked fashion, and we analyzed the data from only Trials 11–110 to avoid including a boredom or fatigue effect that was apparent in the latter half segment of the data collected for each condition. We excluded the first 10 trials to eliminate trials when the participants were getting familiar with each experimental condition. We randomly determined the order of the task demands of the 11 conditions at the given amplitude for each participant. The target width in every condition was fixed at 1.8 mm. Thus, the task-defined index of difficulty (*ID*; Fitts, 1954) with 100 mm in distance was 6.8 bits. We conducted testing in two experimental sessions with five or six movement time conditions per day. We analyzed only the data recorded in the primary direction of motion (*x*-axis) because that method creates the largest error and parallels what is reported in the existing literature (e.g., Fitts, 1954; Schmidt et al., 1979).

Data Analysis

The independent variable was movement time, whereas the primary dependent variables in the experiment were the information entropy of movement outcome (hereafter called *outcome entropy*), constant error (*CE*), variable error (*VE*), and coefficient of variation (*SD/M*, i.e., *CV*) of spatial and temporal errors. The *CV* is a dimensionless index of movement variability (Hancock & Newell, 1985); it expresses the *SD* as a ratio of the mean value, which allows one to compare the variability of different variables (Croxton & Cowden, 1955; Norušis, 2000). For each condition, we calculated average movement velocity by dividing mean movement amplitude by mean movement time.

To calculate the outcome entropy of discrete aiming movements, one needs to know the probabilities of the data distribution of the movement outcome. In the method for obtaining the probabilities in the experimental data, we used properties of the actual frequency distribution, and we calculated the entropy (H_P) obtained with the following equation:

$$H_P = \sum P_i \log_2(1/P_i), \qquad (1)$$

where P_i is defined in the frequency distribution, indicating the relative frequencies of data points in the *i*th bin (Shannon & Weaver, 1949; Williams, 1997). Lai et al. (2005) conducted a preliminary investigation of information entropy in discrete aiming movements. They showed that the entropy H_P that was calculated with no assumption on the nature of the data distribution and the entropy H_G that was calculated on the assumption that the data had a normal distribution produced different results in the estimate of information entropy, indicating that the movement outcome data were not from a normal distribution.

To calculate information entropy on the basis of the frequencies of the data in different bins, one has to relate the number of bins needed in H_P to the standard deviation (σ) used in H_G. The σ controls the spread of a normal distribution, and a spread of 6 σs can approximately cover the whole data range of the normal distribution at the movement outcome. It should be noted that the general expression (Equation 1) for Shannon and Weaver's (1949) entropy does not contain the standard deviation σ explicitly but only the bin frequencies P_i.

For the H_P, furthermore, the dimensionless measured unit and an appropriate bin size compared with the standard deviation σ used in H_G are the two criteria one uses to determine the number of bins to be used in the H_P formula. One can derive an adjusted H_P on the basis of Equation 1; the adjusted H_P provides an estimate of the absolute entropy (Lai et al., 2005), as follows:

$$H_P\,(\sigma, N, R) = H_P + \log_2(\sigma) - \log_2(N, R), \qquad (2)$$

where N is the number of bins and R is the relative range of the data distribution at the movement outcome. For our data, we set $N = 20$ and $R = 6$ (6 σs in a normal distribution) so that we could avoid in our samples (100 trials) extreme cases in which all data points fall into the same bin or each bin contains only zero or one observation. With that adjusted H_P equation, one can compare H_P with H_G. The full details of the entropy algorithms can be found in Lai et al. (2005).

We used a one-way repeated-measures analysis of variance (ANOVA) to test the effect of movement time on movement outcome entropy, *CE*, *VE*, skewness, kurtosis, and *CV* variables. We determined the effect size of all sources by calculating eta square (η^2; Green & Salkind, 2003); an η^2 value greater than 0.14 is considered to reflect a large effect size. We set an alpha level of .05 as the critical level to avoid Type I error in all statistical analyses. To test current notions on the speed–accuracy relation outlined in the introduction, we used Sigma Plot 2000 to fit the *VE*, *CV*, and entropy values as a function of average velocity with a sigmoidal function (three parameters), $y = a/\{1 + e^{-[(x-y_0)/b]}\}$; a logarithmic function (three parameters), $y = y_0 + a\ln(x - x_0)$; and a linear function, $y = y_0 + ax$. We analyzed the peak trajectory variability, the highest maximum entropy value of the trajectory of discrete aiming movements.

Results and Discussion

Spatial Error

Constant Spatial Error

The mean constant spatial error (CE_{space}) as a function of average movement velocity is shown in the upper-left panel in Figure 1.[1] A significant effect of movement time was observed on CE_{space}, $F(10, 70) = 3.21$, $p < .01$, $\eta^2 = 0.31$. The mean constant error stayed around zero in the relatively longer movement time conditions and then increased progressively to overshoot the target with increments in move-

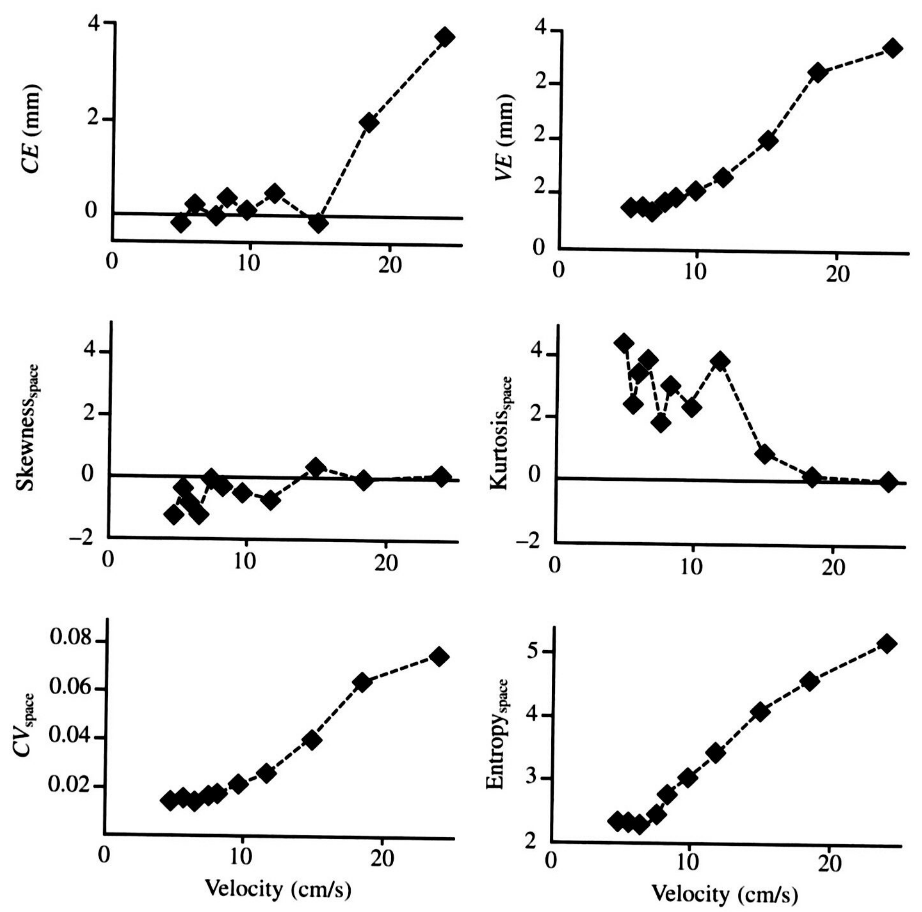

FIGURE 1. Illustrations of six spatial errors: constant spatial error (*CE* in mm), variable spatial error (*VE* in mm), skewness, kurtosis, coefficient of variation in space (*CV*), and outcome entropy in space (Entropy).

ment velocity above 15 cm/s. The analysis of Tukey's Honestly Significant Difference (HSD) Test revealed that the CE_{space} of the 300-ms condition differed significantly from that of 475-ms condition. Other significant differences of CE_{space} were found between 1,175-ms and 1,875-ms conditions, and between 1,175-ms and 2,050-ms conditions.

Variable Spatial Error

The upper-right panel in Figure 1 illustrates variable spatial error (VE_{space}) as a function of average movement velocity. There was a significant effect of movement time on VE_{space}, $F(10, 70) = 81.49$, $p < .01$, $\eta^2 = 0.92$. VE_{space} increased systematically with increases of movement velocity. Tukey's HSD test showed that the *VE*s of the relatively fast movement time conditions (300, 475, and 650 ms) were each different from the *VE*s of other movement time conditions.

The R^2 values of sigmoidal, logarithmic, and linear functions for *VE*, *CV*, and entropy in spatial and temporal dimensions are shown in Table 1. A one-way ANOVA showed that the effect of the R^2 function was significant for VE_{space}, $F(2, 14) = 11.61$, $p < .01$, $\eta^2 = 0.62$. Post hoc analysis showed that the sigmoidal R^2 values were larger than the logarithmic R^2 values.

Skewness of Spatial Error

The effects of movement time on skewness of spatial error ($Skewness_{space}$) were statistically significant, $F(10, 70) = 2.78$, $p < .01$, $\eta^2 = 0.28$. Tukey's HSD test revealed that the $Skewness_{space}$ of the 300-, 475-, and 650-ms conditions differed significantly from those of the 1,525-, 1,700-, and 2,050-ms conditions.

Kurtosis Spatial Error

There were significant effects of movement time on kurtosis of spatial error ($Kurtosis_{space}$), $F(10, 70) = 2.17$, $p < .05$, $\eta^2 = 0.24$. Post hoc analysis revealed that the relatively fast movement time conditions (300 and 475 ms) differed significantly from the relatively moderate and slow movement time conditions (1,175, 1,350, 1,525, 1,700, 1,875, and 2,050 ms).

CV Spatial Error

The *CV* spatial error (CV_{space}) is represented in the bottom-left panel in Figure 1. The pattern of the spatial error changes over average movement velocity seems to be S-shaped. Table 1 shows the R^2 values of sigmoidal, logarithmic, and linear functions for CV_{space}. The sigmoidal func-

TABLE 1. R^2 of Variable Error (*VE*) and Coefficient of Variation (*CV*), Entropy of Spatial and Temporal Error Over Average Movement Velocity for Individual Participants, Average (*M*) of the Individual Data, and Averaged Error Functions for Experiment 1

	VE			*CV*			Entropy		
Participant	sig 3	log 3	linear	sig 3	log 3	linear	sig 3	log 3	linear
				Distance					
P1	.98	.91	.91	.98	.91	.92	.97	.96	.96
P2	.93	.91	.91	.96	.92	.93	.76	.76	.72
P3	.93	.91	.92	.93	.92	.92	.93	.92	.92
P4	.99	.98	.98	.99	.98	.98	.97	.97	.97
P5	.89	.79	.78	.88	.79	.77	.95	.94	.91
P6	.93	.88	.88	.93	.88	.88	.77	.74	.74
P7	.97	.96	.96	.97	.96	.96	.86	.86	.86
P8	.80	.74	.75	.83	.75	.77	.71	.65	.66
M of P1–P8	.93	.89	.89	.93	.89	.89	.87	.85	.84
M of data	.98	.95	.96	.98	.96	.96	.99	.98	.98
				Time					
P1	.88	.91	.84	.96	.96	.94	.79	.84	.79
P2	.89	.86	.87	.89	.85	.61	.79	.82	.79
P3	.67	.84	.59	.85	.80	.81	.39	.75	.38
P4	.83	.84	.79	.79	.79	.74	.84	.85	.84
P5	.65	.80	.60	.63	.61	.58	.66	.71	.65
P6	.88	.92	.84	.89	.88	.86	.85	.92	.85
P7	.74	.82	.70	.92	.92	.91	.72	.76	.71
P8	.63	.72	.58	.92	.89	.89	.54	.66	.54
M of P1–P8	.77	.84	.73	.86	.84	.79	.70	.79	.69
M of data	.91	.97	.85	.99	.98	.97	.89	.98	.88

Note. sig 3 = sigmoidal, 3 parameters; log 3 = logarithmic, 3 parameters; and linear = polynominal linear.

tion fit best for the individual data, the average of the individual data, and the averaged data. An ANOVA and Tukey's HSD analysis on the R^2 values indicated that the R^2 value of the sigmoidal function, $F(2, 14) = 12.57$, $p < .01$, $\eta^2 = 0.64$, was significantly higher than the R^2 values of the logarithmic and the linear functions.

We contrasted the three functions fitted with Akaike's information criterion corrected for small sample sizes (AICc; Burnham & Anderson, 2002). We used the AICc to test how well the functions fit the CV_{space} data compensating for the number of parameters in each equation. In that evaluation approach, the lower the AICc, the better fit to the function. The AICc values were −94.45, −84.08, and −88.88 for the sigmoidal, logarithmic, and linear functions, respectively. The results of that analysis provided support for the proposition that the CV_{space} data were S-shaped.

Information Entropy (Spatial Outcome Entropy)

The general increase of spatial outcome entropy ($Entropy_{space}$) as a function of average movement velocity (cm/s) is illustrated in the bottom-right panel of Figure 1. The significant effect of movement time was observed on $Entropy_{space}$, $F(10, 70) = 32.28$, $p < .001$, $\eta^2 = 0.82$. The analysis of Tukey's HSD test revealed that the $Entropy_{space}$ of the 11 movement time conditions differed significantly between (a) the 300-ms condition and the other conditions, (b) 475 ms and the other conditions (except for the 650-ms condition), and (c) the 650 ms condition and the other conditions (except for the 475- and the 825-ms conditions).

The effect of the R^2 functions for $Entropy_{space}$ was significant, $F(2, 14) = 5.58$, $p < .01$, $\eta^2 = 0.44$. Post hoc analysis showed that the sigmoidal R^2 value was larger than the linear R^2 value.

Temporal Error

Constant Temporal Error

Constant temporal error (CE_{time}) as a function of average movement velocity is shown in the upper-left panel of Figure 2. There was a significant effect of movement time on CE_{time}, $F(10, 70) = 5.20$, $p < .01$, $\eta^2 = 0.43$. CE_{time} was less than zero to an undershooting state in the longest movement time conditions, and increased progressively as the movement time decreased (i.e., as the movement velocity

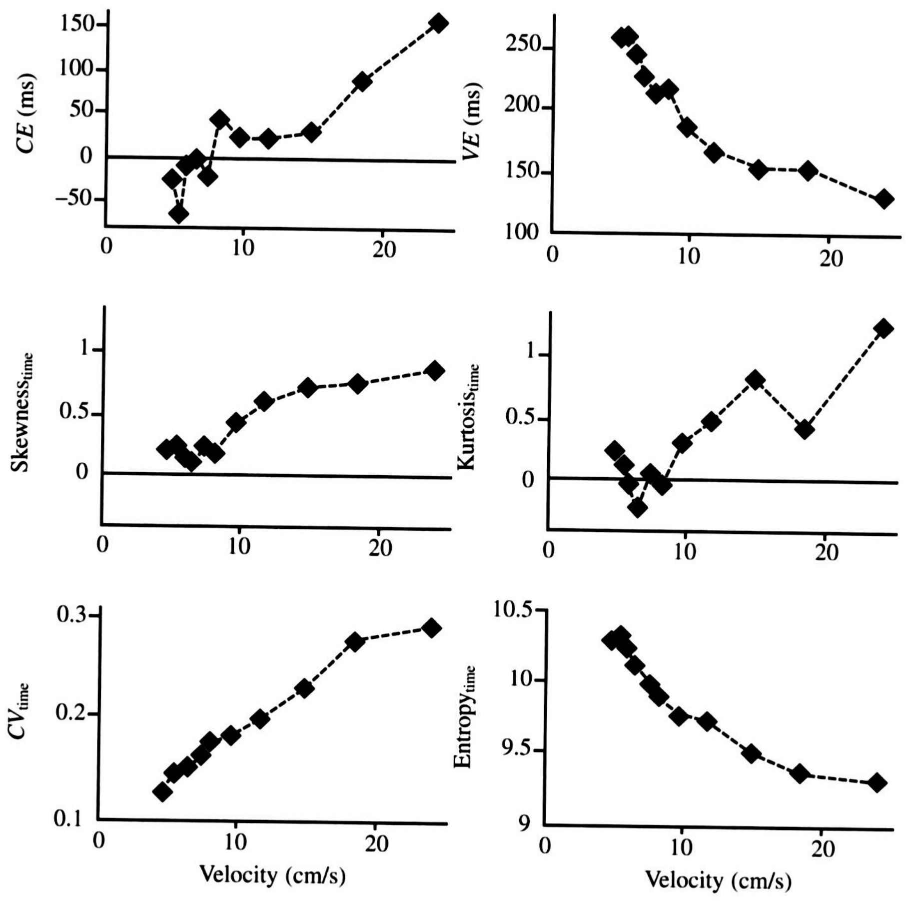

FIGURE 2. Illustrations of six temporal errors: constant temporal error (*CE* in ms), variable temporal error (*VE* in ms), skewness, kurtosis, coefficient of variation in time (*CV*), and outcome entropy in time (Entropy).

increased, particularly above 15 cm/s). Tukey's HSD post hoc analysis showed that the CE_{time} of relatively fast movement time conditions (300 and 475 ms) differed significantly from the CE_{time} of the relatively slow movement time conditions (1,350, 1,525, 1,700, 1875, and 2,050 ms).

Variable Temporal Error

The upper-right panel of Figure 2 illustrates variable temporal error (VE_{time}) as a function of average movement velocity. There was a significant effect of movement time on VE_{time}, $F(10, 70) = 21.91$, $p < .01$, $\eta^2 = 0.76$. VE_{time} decreased progressively with increments of average movement velocity. The analysis of Tukey's HSD test again showed that the *VE*s of the 300-, 475-, 650-, 875-, and 1,000-ms conditions were individually significantly different from the *VE*s of 1,175-, 1,350-, 1,525-, 1,700-, 1,875-, and 2,050-ms conditions.

The R^2 values of sigmoidal, logarithmic, and linear functions for VE_{time} differed significantly, $F(2, 14) = 12.81$, $p < .01$, $\eta^2 = 0.65$; the main difference occurred between the linear and sigmoidal functions and between the linear and logarithmic functions. The sigmoidal R^2 value had the highest percentage of variance of the three functions.

Skewness Temporal Error

There were significant effects of movement time on skewness of temporal error ($Skewness_{time}$), $F(10, 70) = 5.98$, $p < .01$, $\eta^2 = 0.46$. Tukey's HSD test analysis again showed that the $Skewness_{time}$ of relatively fast movement time conditions (300, 475, 650, and 825 ms) was significantly different from that of the relatively slower movements (1,525, 1,700, 1,875, and 2,050 ms).

Kurtosis Temporal Error

There was no significant effect of movement time on Kurtosis temporal error ($Kurtosis_{time}$), $F(10, 70) = 1.86$, $p > .05$.

CV Temporal Error

The *CV* of temporal error (CV_{time}; see Table 2 and bottom-left panel of Figure 2) over increments of average movement velocity was again S-shaped. The sigmoidal function fit best for the individual data, the average of the individual data, and the averaged data. An ANOVA on the R^2 value of the sigmoidalfunction, $F(2, 14) = 3.49$, $p < .05$, $\eta^2 = 0.33$, was significantly higher than were the ANOVAs on the R^2 values of logarithmic and linear functions. The analysis of Tukey's HSD test revealed that the three function fits differed significantly from each other and that the sigmoidal function accounted for the largest percentage of variance. We again contrasted the three functions by using AICc (Burnham & Anderson, 2002). As mentioned earlier, the lower the AICc, the better fit the function. The AICc values were −79.63, −77.22, and −73.21 for the sigmoidal, logarithmic, and linear functions, respectively. That finding confirmed that the S-shaped curve was the robust fit for the CV_{time}.

Information Entropy (Temporal Entropy)

Temporal outcome entropy ($Entropy_{time}$) as a function of average movement velocity (cm/s) is shown in the bottom-right panel of Figure 2. There was a significant effect of movement time on $Entropy_{time}$, $F(10, 70) = 18.47$, $p < .01$, $\eta^2 = 0.73$. The analysis of Tukey's HSD test revealed that the $Entropy_{time}$ of the 11 movement time conditions differed significantly from each other. The values of $Entropy_{time}$ decreased as the average movement velocity increased, however, unlike the values of $Entropy_{space}$. That result is consistent with the proposition that there is complementarity between spatial and temporal outcome entropies and, furthermore, that there is a space–time variability tradeoff in discrete aiming movements (Hancock & Newell, 1985; Newell, 1980). The R^2 functions for $Entropy_{time}$ were significant, $F(2, 14) = 5.30$, $p < .05$, $\eta^2 = 0.43$. The main difference occurred between logarithmic and linear functions. The logarithmic R^2 values accounted for more of the variance than did the linear R^2 values.

Movement Trajectory Variability (Information Entropy)

The change of the trajectory variability of discrete aiming movements showed an inverted U-shape from beginning to end of movement. In the current analysis, we focused only on the occurrence of maximum entropy of the discrete aiming movements. We also conducted a skewness analysis to examine the data distribution at each time point of the movement trajectory.

Occurrence of Maximum Entropy

Figure 3 depicts the occurrence of maximum entropy in terms of percentage of movement time as a function of average movement velocity (cm/s). We conducted a one-way ANOVA to evaluate the relationship between movement time and the occurrence of peak entropy, a relationship that characterizes the essence of maximal trajectory variability of discrete aiming movements. The independent variable, movement time, included 11 different levels ranging from 300 ms to 2,050 ms. The dependent variable was the occurrence of maximum entropy as a percentage of the time into the movement trajectory. The result of the ANOVA was significant, $F(10, 70) = 2.48$, $p < .05$, showing that movement time and, hence, movement velocity affected the relative time of peak entropy in the movement trajectory. Tukey's HSD post hoc analysis revealed that the significant difference occurred between the fastest movement time condition (300 ms) and all other movement time conditions.

The analysis of Pearson product-moment correlation coefficient over all conditions provided evidence that the correlation between the maximum entropy in the trajectory and the outcome entropy was significant, $r(88) = .32$, $p < .01$. The percentage of variance accounted for in the relation was very small (~10%). Moreover, only 1 of the 11 individual conditions produced a significant correlation between those variables. In addition, the values of maximal trajectory variability

TABLE 2. R^2 **of Variable Error (*VE*) and Coefficient of Variation (*CV*), Entropy of Spatial and Temporal Error Over Average Movement Velocity for Individual Participants, Average (*M*) of the Individual Data, and Averaged Error Functions for Experiment 2, 300-ms Condition**

	VE			*CV*			Entropy		
Participant	sig 3	log 3	linear	sig 3	log 3	linear	sig 3	log 3	linear
				Space					
P1	.91	.88	.89	78	.85	.75	.98	.97	.91
P2	.94	.92	.83	89	.93	.84	.75	.75	.75
P3	.88	.89	.88	96	.99	.93	.94	.94	.70
P4	.88	.86	.87	91	.91	.86	.93	.94	.93
P5	.99	.97	.90	.96	.92	.92	.98	.96	.80
P6	.93	.91	.83	.87	.83	.84	.98	.97	.68
P7	.80	.80	.79	.91	.92	.88	.93	.92	.60
P8	.81	.79	.79	.84	.91	.82	.79	.78	.76
M of P1–P8	.89	.88	.85	.89	.91	.86	.91	.90	.77
M of data	.96	.96	.94	.98	.99	.97	.99	.98	.88
				Time					
P1	.18	.17	.18	.62	.61	.61	.21	.18	.21
P2	.06	.60	.07	.20	.38	.20	.18	.67	.17
P3	.05	.06	.05	.06	.13	.07	.11	.11	.11
P4	.08	.07	.08	.24	.23	.24	.36	.48	.34
P5	.63	.48	.49	.83	.75	.76	.96	.96	.92
P6	.06	.54	.05	.06	.12	.12	.06	.11	.12
P7	.05	.05	.06	.32	.30	.31	.86	.49	.50
P8	.63	.72	.58	.92	.89	.89	.54	.66	.54
M of P1–P8	.23	.30	.17	.41	.42	.40	.38	.42	.34
M of data	.49	.49	.50	.87	.80	.80	.80	.49	.50

Note. sig 3 = sigmoidal, 3 parameters; log 3 = logarithmic, 3 parameters; and linear = polynominal linear.

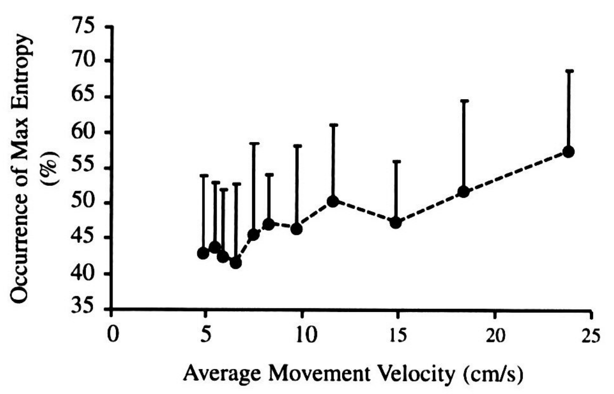

FIGURE 3. Occurrence of maximum entropy (%) as a function of average movement velocity (cm/s). Error bars represent 95% confidence interval for mean percentage.

(4.84–6.68 bits; figures not shown here) in all movement time conditions across all the participants were lower than the task-defined *ID* (6.80 bits) in the 100-mm condition. For example, the average peak entropy values for the 11 time conditions (from 300 to 2,050 ms) were 6.14, 5.93, 5.67, 5.72, 5.72, 5.60, 5.62, 5.66, 5.79, 5.71, and 5.54 bits, respectively.

Skewness Analysis of Movement Data Distribution

In Figure 4, we present examples of the average skewness of data distribution as a function of time from two extreme movement spatiotemporal conditions, the 100-mm/300-ms condition and the 100-mm/2,050-ms condition. The skewness analysis indicated a strong decreasing skewness regardless of spatial and temporal constraints. All 11 spatiotemporal conditions showed the trend of zero crossing with decreasing skewness (signed from positive to negative values). Figure 4 also shows high skewness for movement outcome in the very slow 100-mm/2,050-ms condition.

Relation of VE *and Entropy*

The *VE* and entropy functions shown in Figure 1 suggested similar properties. To examine the possible distinction between the distributional (*VE*) and probabilistic (entropy) approaches, we applied a Pearson correlation analysis separately on *VE* and entropy in both spatial and temporal aspects for individual participants. The averaged R^2s between spatial *VE* and entropy and between temporal

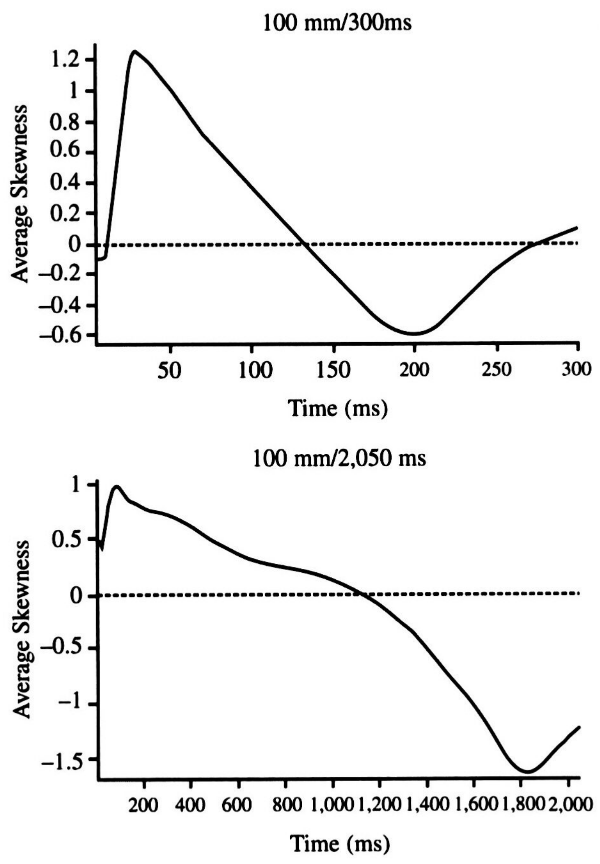

FIGURE 4. Average skewness of data distribution as a function of time from two extreme movement spatiotemporal conditions (100-mm/300-ms and 100-mm/2,050-ms conditions).

VE and entropy were .84 (max = .94 and min = .72) and .72 (max = .89 and min = .52), respectively.

Summary

There were three general findings from Experiment 1. First, the *VE* and *CV* data for spatial error were best fit by a sigmoidal function, and similar, although nonsignificant trends were present for temporal errors. Second, the functions for *VE* and entropy over average movement velocity showed generally similar trends but clearly did not reflect the same properties, as indexed by their modest correlation. Third, the difference between the *VE* and entropy functions was a consequence of the departure from a normal distribution and the systematic change in skewness and kurtosis over the range of movement velocities.

EXPERIMENT 2

In Experiment 2, we contrasted the distributional and entropy approaches to movement error in conditions with constant movement time (MT) but with varied movement distances. We chose a relatively longer MT (800 ms) and a relatively shorter MT (300 ms) on the basis of their differential involvement of visual feedback for limb control (Carlton, 1992). We focused in this experiment on the examination of the speed–accuracy functions for both spatial and temporal movement errors with relatively longer and shorter MT constraints. We again investigated in this experiment the complementarity of the spatial and temporal errors.

Method

Participants

Experiment 2 included the same 6 individuals who had participated in Experiment 1, plus another 2 participants (P3 and P6 in Tables 2 and 3) who had experience from an earlier study (Lai et al., 2005). Thus, there were 8 experienced participants in this experiment. Their age range was the same as in Experiment 1 (24–31 years; $M = 28.84$ years, $SD = 2.12$ years). They were also given a small monetary reward for their participation.

Apparatus

The apparatus was the same as that used in Experiment 1.

Experimental Design

There were two MTs in the present experiment. In the relatively longer MT condition (800 ms), there were six movement distances: 40, 80, 120, 160, 200, and 240 mm; the corresponding average movement velocities were 5, 10, 15, 20, 25, and 30 cm/s. In the relatively shorter MT condition (300 ms), the six movement distances were 15, 24, 33, 42, 51, and 60 mm; their corresponding average movement velocities were 5, 8, 11, 14, 17, and 20 cm/s. According to a pilot study, the given 300-ms condition was approximately the fastest time that participants could perform under the limitations of the current equipment and task constraints, especially for the 60-mm condition. We assumed that there was a more limited role for visual feedback control in the 60-mm/300-ms condition movement but that there was more involvement of visual feedback control in the 800-ms condition movements. We asked participants to perform 100 trials in each space–time condition.

Data Analysis

We used the individual one-way repeated-measures ANOVA (in the 300- and the 800-ms conditions) to test the effect of movement distance on movement *CE*, *VE*, skewness, kurtosis, *CV*, and outcome entropy. We also used a one-way ANOVA to examine the effect of movement distance on the relative time of maximum entropy in the movement trajectory. We fit the *VE*, *CV*, and entropy data as a function of average velocity with the same three functions used in Experiment 1, that is, sigmoidal, logarithmic, and linear.

Results and Discussion

Spatial Error

Figure 5 illustrates six different spatial errors as a function of average movement velocity and MT condition: *CE*,

TABLE 3. R^2 of Variable Error (*VE*) and Coefficient of Variation (*CV*), Entropy of Spatial and Temporal Error Over Average Movement Velocity for Individual Participants, Average (*M*) of the Individual Data, and Averaged Error Functions for Experiment 2, 800-ms Condition

	VE			*CV*			Entropy		
Participant	sig 3	log 3	linear	sig 3	log 3	linear	sig 3	log 3	linear
				Space					
P1	.93	.95	.95	.28	.16	.17	.89	.94	.78
P2	.85	.83	.83	.05	.21	.15	.97	.93	.82
P3	.74	.72	.72	.04	.33	.04	.89	.88	.89
P4	.96	.91	.73	.91	.69	.70	.97	.95	.83
P5	.91	.85	.85	.10	.56	.09	.86	.84	.84
P6	.96	.90	.87	.14	.09	.04	.94	.93	.87
P7	.94	.85	.79	.04	.08	.04	.95	.91	.85
P8	.91	.88	.90	.66	.64	.65	.97	.96	.96
M of P1–P8	.90	.86	.83	.28	.35	.24	.93	.92	.86
M of data	.96	.96	.96	.73	.62	.63	.99	.98	.96
				Time					
P1	.21	.77	.20	.53	.68	.52	.04	.40	.02
P2	.04	.58	.59	.66	.81	.66	.76	.83	.77
P3	.60	.92	.59	.66	.86	.65	.77	.80	.78
P4	.43	.41	.41	.68	.57	.58	.05	.25	.25
P5	.04	.38	.04	.13	.68	.12	.04	.41	.04
P6	.05	.09	.04	.06	.10	.05	.02	.02	.02
P7	.05	.24	.05	.12	.02	.03	.03	.02	.03
P8	.63	.72	.58	.92	.89	.89	.54	.66	.54
M of P1–P8	.18	.45	.26	.48	.54	.41	.32	.44	.34
M of data	.76	.86	.75	.91	.93	.91	.44	.91	.44

Note. sig 3 = sigmoidal, 3 parameters; log 3 = logarithmic, 3 parameters; and linear = polynominal linear.

VE, skewness, kurtosis, *CV*, and information entropy (Entropy).

CE_{space}

The mean constant spatial errors in the 300- and 800-ms conditions ($CE_{space,300ms}$ and $CE_{space,800ms}$) as a function of average movement velocity are shown in the upper-left panel in Figure 5. There was a significant effect of movement distance on $CE_{space,300ms}$, $F(5, 35) = 4.13$, $p < .01$, $\eta^2 = 0.37$, but not on $CE_{space,800ms}$, $F(5, 35) = 1.37$, $p > .05$. Tukey's HSD test revealed that the *CE* of the 42-mm/300-ms condition differed from all other *CE*s in the 300-ms condition. Only that *CE* exceeded 1.0 mm. We found that the mean *CE*s for the 300- and 800-ms conditions showed a tendency to overshoot the target at all movement velocities, indicating that overshooting is a general phenomenon when a small target is used in aiming tasks (Jastrow, 1886; Proteau & Isabelle, 2002).

VE_{space}

Figure 5 (upper-right panel) shows the variable spatial error in the 300- and 800-ms conditions ($VE_{space,300ms}$ and $VE_{space,800ms}$). The effects of movement distance in both *VE*s were statistically significant: For $VE_{space,300ms}$, $F(5, 35) = 38.75$, $p < .001$, $\eta^2 = 0.85$; for $VE_{space,800ms}$, $F(5, 35) = 38.48$, $p < .001$, $\eta^2 = 0.85$. Generally, *VE* increased as a function of movement velocity. Tukey's HSD test showed that all *VE*s differed from each other, except the *VE*s among 15, 24, and 33 mm in the 300-ms condition.

Tables 2 (300-ms condition) and 3 (800-ms condition) show the R^2 values of sigmoidal, logarithmic, and linear functions for *VE*, *CV*, and entropy in spatial and temporal dimensions. A one-way repeated-measures ANOVA showed that the effect of the R^2 functions was significant for both $VE_{space,300ms}$, $F(2, 14) = 6.13$, $p < .05$, $\eta^2 = 0.47$, and $VE_{space,800ms}$, $F(2, 14) = 4.77$, $p < .05$, $\eta^2 = 0.41$. For both MT conditions, the R^2 values were higher for the sigmoidal than for the logarithmic function.

$Skewness_{space}$

For the 300-ms MT conditions there was a significant effect of movement distance on skewness of spatial error, $F(5, 35) = 4.21$, $p < .01$, $\eta^2 = 0.38$. There was no effect on skewness in the 800-ms condition. Tukey's HSD test revealed that the spatial skewness of the 15-mm/300-ms condition differed significantly from the spatial skewness of

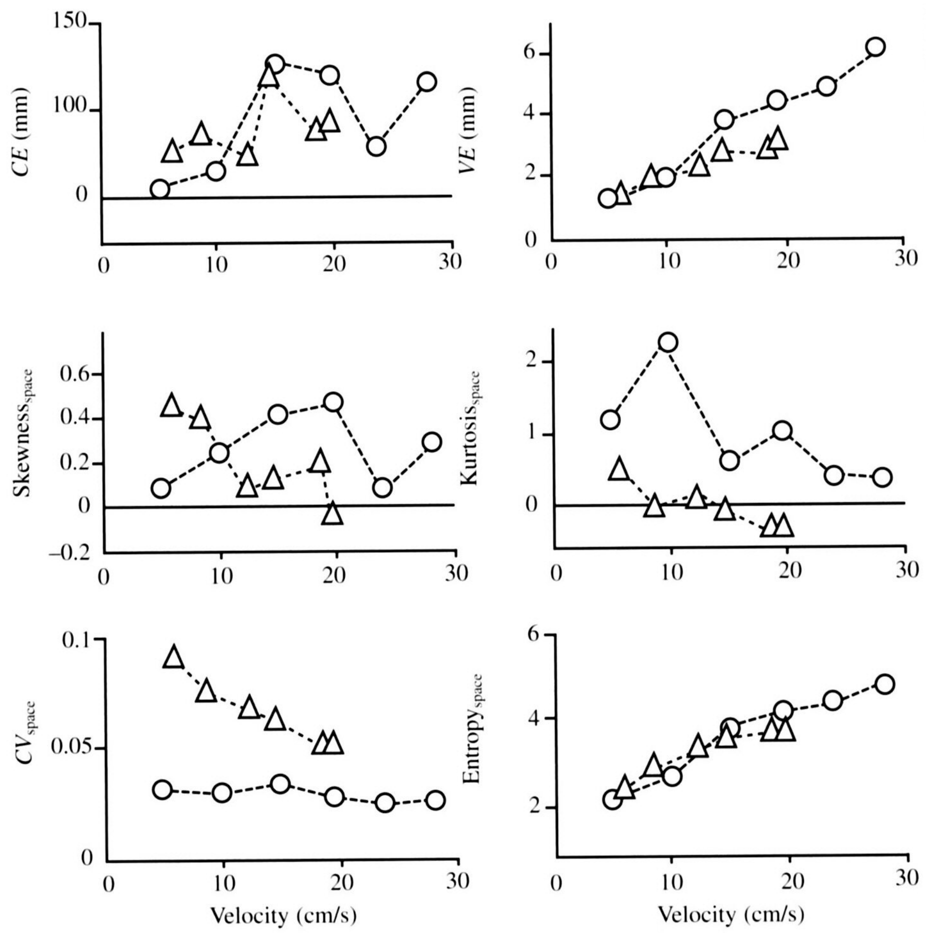

FIGURE 5. Illustrations of six spatial errors: constant spatial error (*CE* in mm), variable spatial error (*VE* in mm), skewness, kurtosis, coefficient of variation in space (*CV*), and outcome entropy in space (Entropy). Open triangles indicate the 300-ms condition, and open circles indicate the 800-ms condition.

the 60-mm/300-ms condition, and that the spatial skewness of the 24-mm/300-ms condition differed from those of the 33-mm/300-ms, 42-mm/300-ms, 51mm/300-ms, and the 60-mm/300-ms conditions.

Kurtosis$_{space}$

There was no significant effect of MT on kurtosis of spatial error for the 300-ms and the 800-ms MT conditions.

CV$_{space}$

The spatial coefficients of variation in the 300- and 800-ms conditions ($CV_{space,300ms}$ and $CV_{space,800ms}$) are shown in the bottom-left panel in Figure 5. The effect of movement distance was significant on $CV_{space,300ms}$, $F(5, 35) = 30.72$, $p < .001$, $\eta^2 = 0.81$, but not on $CV_{space,800ms}$, $F(5, 35) = 2.08$, $p > .05$. The relative spatial variability decreased with movement velocity only in the very short duration, 300-ms condition; in the 800-ms condition, there was little change in relative spatial variability.

With regard to the relationship between *CV*s and velocity, the effect of the R^2 functions was significantly different only in the 300-ms condition ($CV_{space,300ms}$), $F(2, 14) = 9.27$, $p < .01$, $\eta^2 = 0.57$; the main differences occurred between the sigmoidal and linear functions (sigmoidal R^2 > linear R^2) and between the logarithmic and linear functions (logarithmic R^2 > linear R^2; Tables 2 and 3 provide the R^2 values). There was no function effect on $CV_{space,800ms}$, $F(2, 14) = 1.64$, $p > .05$.

Information Entropy (Spatial Entropy)

Figure 5 (bottom-right panel) displays the change of spatial outcome entropy in the 300- and 800-ms conditions (Entropy$_{space,300ms}$ and Entropy$_{space,800ms}$) as a function of average movement velocity (cm/s). The effects of movement distance in both entropies for Entropy$_{space,300ms}$, $F(5, 35) = 28.61$, $p < .001$, $\eta^2 = 0.80$, and for Entropy$_{space,800ms}$, $F(5, 35) = 57.68$, $p < .001$, $\eta^2 = 0.89$, were significant. Tukey's HSD post hoc analysis revealed that in the 300-ms condition significant differences in movement distance occurred between the 15-, 24-, 31-, and 42-mm conditions. There were no differences for the $\text{Entropy}_{space,300ms}$ in the 42-, 51-, and 60-mm conditions. The $\text{Entropy}_{space,800ms}$ at all movement distances were different from each other, and the size of the difference increased as a function of average movement velocity.

A one-way repeated-measures ANOVA showed that the effect of the R^2 curve-fit functions was significant for both $\text{Entropy}_{space,300ms}$, $F(2, 14) = 8.78, p < .01, \eta^2 = 0.56$, and for $\text{Entropy}_{space,800ms}$, $F(2, 14) = 9.31, p < .01, \eta^2 = 0.57$. The difference occurred between the sigmoidal and linear functions for both spatial conditions. The R^2 values of the sigmoidal function were higher than were those of the linear function (see Tables 2 and 3 for the R^2 values).

Temporal Error

Figure 6 illustrates the six different representations of temporal error as a function of average movement velocity and MT condition: *CE*, *VE*, skewness, kurtosis, *CV*, and Entropy_{time}. The open triangles represent the 300-ms condition; the open circles, the 800-ms condition.

CE_{time}

The CE_{time} in the 300- and the 800-ms conditions ($CE_{time,300ms}$ and $CE_{time,800ms}$) as a function of average movement velocity are shown in the upper-left panel in Figure 6. A significant effect of movement distance was found for both *CE*s: $CE_{time,300ms}$, $F(5, 35) = 9.64, p < .001, \eta^2 = 0.58$; $CE_{time,800ms}$, $F(5, 35) = 5.47, p < .001, \eta^2 = 0.44$. For the 300-ms condition, increasing velocity led to temporal undershooting on target time, although for the 800-ms condition, increasing velocity resulted in temporal overshooting on target time. Post hoc analysis showed that the significant differences of $CE_{time,300ms}$ were found individually only between 15- and 60-mm, 24- and 60-mm, and 33- and 60-mm conditions. For the $CE_{time,800ms}$, Tukey's HSD post hoc analysis showed that the *CE*s of the relatively fast movement conditions (51 and 60 mm) differed significantly from those of the relatively slow movement conditions (15, 24, 33, and 42 mm).

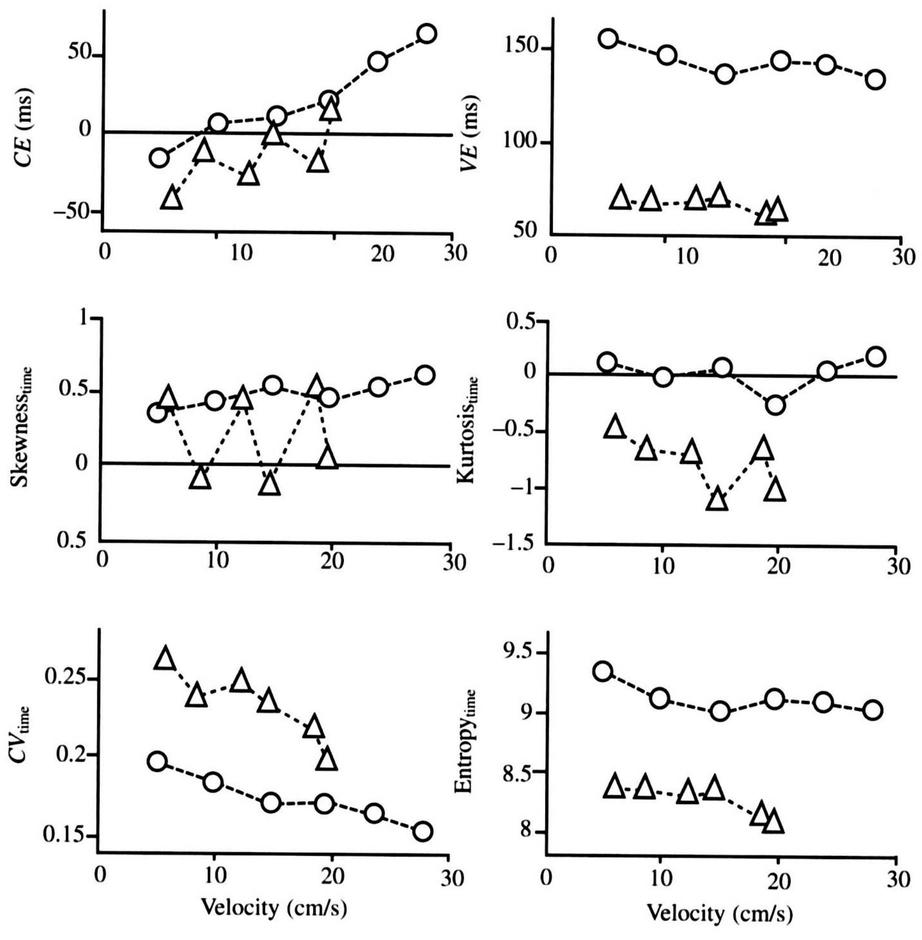

FIGURE 6. Illustrations of six temporal errors: constant temporal error (*CE* in ms), variable temporal error (*VE* in ms), skewness, kurtosis, coefficient of variation in time (*CV*), and outcome entropy in time (Entropy). Open triangles indicate the 300-ms condition, and open circles indicate the 800-ms condition.

VE

The upper-right panel in Figure 6 illustrates changes of VE_{time} in the 300- and 800-ms conditions ($VE_{time,300ms}$ and $VE_{time,800ms}$) over average movement velocity. The effect of movement distance was significant on $VE_{time,300ms}$, $F(5, 35) = 2.55$, $p < .05$, $\eta^2 = 0.27$, but not on $VE_{time,800ms}$, $F(5, 35) = 1.33$, $p > .05$. For the 300-ms condition, a significant difference in movement distance occurred between 15- and 60-mm conditions, indicating the extreme velocity for VE_{time} was a critical factor for the changes of VE in the short duration.

We used the same three R^2 functions (i.e., sigmoidal, logarithmic, and linear) to test the relationship between VE and velocity. The R^2 values of the three functions for VE, CV, and entropy in spatial and temporal aspects are shown in Tables 2 (300-ms condition) and 3 (800-ms condition). The one-way repeated-measures ANOVA showed that the R^2 function was significant only for $VE_{time,800ms}$, $F(2, 14) = 6.56$, $p < .05$, $\eta^2 = 0.48$; the main differences occurred between the sigmoidal and logarithmic functions. The sigmoidal R^2 values were larger than the logarithmic R^2 values (see Tables 2 and 3). There was no function effect for $VE_{time,300ms}$, $F(2, 14) = 1.12$, $p > .05$.

$Skewness_{time}$

For the 300-ms MT conditions, there was a significant effect of movement distance on skewness of temporal error, $F(5, 35) = 7.29$, $p < .01$, $\eta^2 = 0.51$. However, there was no effect of movement distance on skewness of temporal error at the 800-ms condition. The Tukey's HSD post hoc analysis revealed that the $Skewness_{time}$ of the 15-mm/300-ms, 33-mm/300-ms, and 51-mm/300-ms conditions was significantly higher than the temporal skewness of the 24-mm/300-ms, 42-mm/300-ms, and 60-mm/300-ms conditions.

$Kurtosis_{time}$

There was no significant effect of movement distance on $Kurtosis_{time}$ for both the 300-ms and 800-ms MT conditions.

CV

In Figure 6 (bottom-left panel), we present the CV_{time} in the 300- and 800-ms conditions ($CV_{time,300ms}$ and $CV_{time,800ms}$). A significant effect of movement distance was found in both CVs; for $CV_{time,300ms}$, $F(5, 35) = 4.78$, $p < .01$, $\eta^2 = 0.41$, and for $CV_{time,800ms}$, $F(5, 35) = 3.71$, $p < .01$, $\eta^2 = 0.35$. Tukey's HSD post hoc analysis revealed that in both CVs, the significant difference in movement distance occurred between the 15- and 60-mm conditions. Increasing velocity generally led to the reduction of the relative temporal variability at different rates. The reduction of $CV_{time,300ms}$ was relatively faster than was the reduction of $CV_{time,800ms}$. However, there were no significant differences for the three R^2 functions on the $CV_{time,300ms}$, $F(2, 14) = 0.48$, $p > .05$, or on the $CV_{time,800ms}$, $F(2, 14) = 1.84$, $p > .05$.

Information Entropy ($Entropy_{time}$)

The bottom-right panel in Figure 6 shows the $Entropy_{time}$ in the 300- and 800-ms conditions ($Entropy_{time,300ms}$ and $Entropy_{time,800ms}$) as a function of average movement velocity (cm/s). There were significant effects of movement distance on both entropies; for $Entropy_{time,300ms}$, $F(5, 35) = 5.25$, $p < .01$, $\eta^2 = 0.43$, and for $Entropy_{time,800ms}$, $F(5, 35) = 2.86$, $p < .05$, $\eta^2 = 0.29$. Tukey's HSD post hoc analysis revealed that in the 300-ms condition, the significant difference in movement distance occurred between the four shorter distances (15, 24, 33, and 42 mm) and the two longer distances (51 and 60 mm). In the 800-ms condition, the significant difference occurred between the shortest distance (15 mm) and the five longer distances (24, 33, 42, 51, and 60 mm). Those findings suggest that although increasing velocity resulted in the reduction of both temporal entropies, the critical velocity thresholds for the movement temporal uncertainty were different in the relatively long (800 ms) and short (300 ms) duration conditions. Relatively fast movements and relatively slow movements caused more reduction of temporal uncertainty in 300- and 800-ms conditions, respectively.

The effect of the R^2 functions was significant only for $Entropy_{time,800ms}$, $F(2, 14) = 3.66$, $p < .05$, $\eta^2 = 0.34$. The Tukey's HSD post hoc analysis revealed that the logarithmic R^2 value was significantly larger than the sigmoidal R^2 value. There was no R^2 function effect for $Entropy_{time,300ms}$, $F(2, 14) = 0.60$, $p > .05$.

Occurrence of Maximum Entropy

Figure 7 shows the occurrence of maximum entropy in terms of percentage of MT as a function of average movement velocity (cm/s) in the 300- and 800-ms conditions. The ANOVA revealed that the effect of movement distance on the occurrence of maximum entropy in the 800-ms condition was significant, $F(5, 35) = 6.87$, $p < .001$, $\eta^2 = 0.50$, but the movement-distance variable had no effect on the occurrence of maximum entropy in the 300-ms condition,

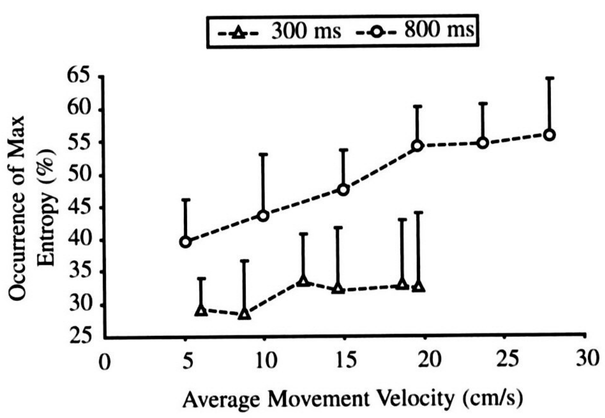

FIGURE 7. Occurrence of maximum entropy (%) as a function of average movement velocity (cm/s).

$F(5, 35) = 2.34$, $p > .05$. The Tukey's HSD post hoc analysis showed that in the 800-ms condition, the relative time of maximum entropy was not different in the three longer distances (160, 200, and 240 mm); however, there were significant differences between the three shorter distances (40, 80, and 120 mm) and each of the three longer distances (160, 200, and 240 mm). Furthermore, only the maximum entropy at the 160-, 200-, and 240-mm amplitudes of the 800-ms MT condition exceeded 50% of the duration of the movement trajectory.

The analysis of Pearson product-moment correlation coefficient revealed that the correlation between the maximum entropy in the trajectory and outcome entropy was significant for both the 300-ms and the 800-ms conditions, $r(48) = .73$, $p < .01$, and $r(48) = .81$, $p < .01$, respectively. The amount of variance accounted for was modest but higher than that shown in Experiment 1. None of the individual movement conditions showed a significant correlation.

The present findings confirmed that the maximum entropy occurs relatively earlier in slower movements. For example, the relatively higher velocity in the 800-ms MT condition led maximum entropy to occur closer to the middle of the duration of the movement trajectory. That is, the peak entropy in the long duration (800-ms) MT occurred at 39.46%, 43.44%, 47.43%, 54.14%, 54.38%, and 55.47% of the movement trajectory in the 40-, 80-, 120-, 160-, 200-, and 240-mm conditions, respectively. Although the movement velocity for the short duration (300 ms) MT showed no significant difference in the occurrence of the peak entropy, the peak entropy systematically occurred at 29.17%, 28.54%, 33.54%, 32.29%, 32.92%, and 32.50% of the temporal duration of the movement trajectory in the 15-, 24-, 33-, 42-, 51-, and 60-mm conditions, respectively.

Relation of *VE* and Entropy

The data in the 300-ms MT condition showed that the averaged R^2 for the individual participants between VE_{space} and Entropy and between VE_{time} and Entropy were .82 (max = .98; min = .48) and .57 (max = .85, min = .14), respectively. For the longer 800-ms MT condition, the averaged R^2s were .89 (max = .97, min = .70) and .55 (max = .87, min = .19) in its spatial and temporal aspects, respectively.

GENERAL DISCUSSION

Entropy is related to the concepts of uncertainty, variability, and probability. The fundamental principle of information theory is that the equiprobable and independent elementary events result in the maximum entropy state (Gatlin, 1972; Shannon & Weaver, 1949). In the field of motor control, information entropy has been expressed as the probable capacity to store and process sets of motor commands from the central nervous system (Fitts, 1954). Information entropy provides an alternative and direct analysis of the uncertainty of movement outcome, in contrast to a Gaussian distributional account of the amount of variability of spatial and temporal errors based on the index of standard deviation (Lai et al., 2005). Entropy also provides a common information dimension on a single index of movement uncertainty, as opposed to a set of dependent variables that are derived from the moments of a distribution (e.g., mean, standard deviation, and skewness). In this study, we examined (a) the function of entropy in different movement speed and accuracy conditions and (b) whether that single entropy measure would reveal a pattern different from that of variable error in movement speed–accuracy conditions.

Entropy and Movement Outcome Variability

Our central objectives in this study were to contrast the entropy and distributional approaches to the analysis of the outcome variability of discrete aiming movements and to determine the spatial and temporal variability and entropy functions. In the distributional approach, one uses the dimensionless index of movement variability, that is, the *CV*, in addition to the standard deviation so that one can contrast movement outcome variability across the scales of error measurement (Hancock & Newell, 1985). The *CV* is also an index that combines the mean and standard deviation, and thus, like the entropy measure, it provides a single index of movement uncertainty.

The *CV* over movement velocity within a given movement amplitude in Experiment 1 produced a sigmoid-like logistic growth curve for both the spatial and temporal error functions. That finding is consistent with the data reported in the foundational study of Woodworth (1899); Woodworth's data reflected an S-shaped function for the *CV* over the velocity range within any given amplitude (see reanalysis in Hancock & Newell, 1985). The present findings also provided additional evidence—here, in an aiming task—that when the measures of spatial and temporal error are considered in the same speed–accuracy movement plane, the movement error functions are isomorphic (Kim et al., 1999). The theorizing of an S-shaped relative variability function (Hancock & Newell) contrasts with the theoretical predictions of the linear (Schmidt et al., 1979), logarithmic (Fitts, 1954), and square root (Meyer et al., 1988) functions for the relation between movement speed and accuracy.

To determine whether the entropy probabilistic estimate of variability produces the same or a different function of the movement speed–accuracy relation than do the distributional approaches to variability, we compared the change in the respective *VE* (spatial or temporal) over movement velocity with the respective change of the entropy measurement. According to the function-fitting comparisons, the speed–accuracy relationsip to MTs over the same distance or the change in average velocity (Experiment 1) generally showed a sigmoid-like function in both *VE* and entropy measures. The *CV*, in particular, is best fit with a sigmoidal function, and there are different directions to the change in variability in space and time consistent with the prediction of Hancock and Newell (1985).

Of central importance, though, to the thrust of this article is that there was only a modest correlation between each of the respective *VE* and entropy measures, although clearly the *VE* function was most closely aligned with the entropy function when we contrasted it with *CE* and *CV*. The difference between the *VE* and entropy measures was enhanced in the longer MT and lower velocity conditions. That finding suggests that the distributional and frequency approaches are driven to a significant degree by changes in different properties of the variability. Furthermore, the difference between the distributional and entropy approaches to variability was magnified in the assessments of the contrast between the individual data and the group average data. That contrast provided further evidence that the assessment of the changes in group data may mask the actual changes that are taking place in individuals.

The analysis of skewness and kurtosis provided evidence that there is a systematic departure from a normal distribution in both spatial and temporal outcome errors (Hancock & Newell, 1985). The outcome data showed trends similar to those found in the analysis of outcome error in a follow-through timing task (Kim et al., 1999). Hancock and Newell hypothesized that the skewness and, to a lesser extent, kurtosis are responsible for the difference in the *VE* and entropy estimates of the speed–accuracy relation over the range of movement conditions. That systematic departure from a normal curve raises interesting issues about the interpretation of *VE* and its centrality in determining speed–accuracy functions.

It is reasonable to assume that when one discusses or advances predictions about variability (*VE*) over movement parameter conditions, the proposal is based on the implicit assumption that the distribution of the movement errors is normal. That assumption raises the following question: What does it formally mean to emphasize or interpret a function for *VE* across parameter changes when there is an accompanying change from a normal distribution in skewness and kurtosis. In contrast, the information entropy approach provides a direct frequency-based analysis that enables one to derive a single score for the uncertainty of the movement outcome. Thus, the formal foundation of probability in the information entropy measure provides a firm foundation for considering the uncertainty of movement error or outcome. Contemporary analyses and theorizing in motor control are so embedded in the background of the central limit theorem and in the standard deviation as the predominant measure of variability that the limitations or problems in the variability *VE* measure usually either are not recognized or are passed over (also see Newell & Slifkin, 1998; Riley & Turvey, 2002).

Meyer et al. (1988) showed that there are different *VE* functions for different kinds of speed–accuracy tasks. It is unknown, however, if and to what degree that assessment is influenced by the different degrees of departure from a normal distribution in different tasks. The generality of the distinction between *VE* and entropy across the different types of movement speed–accuracy tasks requires further investigation. However, those tests must be conducted over a wide range of movement amplitude and time (speed) conditions.

Entropy and Movement Trajectory Variability

We also examined the change of information entropy as a function of amplitude–MT conditions by determining the variability of the trajectory of the discrete aiming movements. We hypothesized that the peak entropy may occur relatively earlier in the long MT (slow movements) conditions because of the stronger involvement of visual feedback in the homing-in phase at the end of the movement (Carlton, 1992; Woodworth, 1899). In that view, the entropy is related to the velocity profile within the movement trajectory and to the timing of the peak velocity within the movement.

The results of Experiment 1 showed that longer MTs led to the occurrence of peak entropy in earlier proportions (in time) of the movement trajectory. Those findings suggest that the fixed movement amplitude (10 cm) in Experiment 1 limited the actual values of maximal trajectory variability in all MT conditions. When we fixed MT and manipulated movement distance (Experiment 2), we also found that the maximum entropy occurred relatively earlier in the slower movements—again, we hypothesize, because of the enhanced role of feedback processes. Consistent with that interpretation, the longer MT led peak entropy to occur proportionally earlier in the movement trajectory.

The analyses also confirmed that the task-defined ID in the Fitts (1954) protocol captures an average entropy, in that the task-defined ID was larger than the peak entropy during the movement. The values of maximal trajectory variability in each MT condition (4.84 to ~6.68 bits/action) were lower than was the task-defined ID (Fitts, 1954) in the 10-cm condition (6.80 bits/action). Thus, the entropy analysis of the trajectory provided a more precise index of movement uncertainty (outcome probability) and afforded a contrast within and between movement trajectories in a single information metric. The analysis of entropy in the trajectory of a Fitts aiming task with a movement target, as opposed to the target point used here, would provide a further test of the effects of task goals on the entropy of the movement and its outcome.

In summary, the variability of movement as indexed by the standard deviation is a biased estimate of the uncertainty of movement outcome when the distribution in question (outcome or trajectory feature, spatial or temporal measure) is not normal. The results of this study have provided further evidence that the error distribution of individual and group-averaged data in speed–accuracy tasks is typically not normal and that the departure from normality changes systematically over the scaling of movement amplitude and time conditions (Hancock & Newell, 1985; Kim et al., 1999). One can interpret variability (*VE*) through the standard deviation only if one has full knowledge of the distributional properties; when the distribution is not normal, as is typically the case in movement speed–accuracy studies, it is a difficult variable to interpret unless one considers other variables. In contrast, information entropy is a single-index representation of the uncertainty of movement outcome based directly on the probabilities of outcome in the movement speed–accuracy relation, and we have shown here that it produces a different function for movement speed and for movement variability.

A determination of whether the single entropy measure is a more useful description of the speed–accuracy relation than is the set of measures of the distribution requires further study over a broader range of conditions than were investigated here. Nevertheless, the entropy interpretation of movement variability in terms of information theory constructs seems, as Fitts (1954) noted, to hold the basis for a general account of uncertainty of movement outcome in both space and time.

NOTE

1. We plotted the data in this figure with velocity on the abscissa because that is the typical manner in which investigators have previously presented that kind of data, even though we calculated the ANOVA statistics with distance and time as the independent variables.

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