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Impulse and Movement Space–Time Variability

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ABSTRACT. In 3 experiments, the authors examined movement space–time variability as a function of the force–time properties of the initial impulse in a movement timing task. In the range of motion and movement time task conditions, peak force, initial rate of force, and force duration were manipulated either independently or in combination across a range of parameter values. The findings showed that (a) impulse variability is predicted well by the elaboration of the isometric force variability scaling functions of L. G. Carlton, K. H. Kim, Y. T. Liu, and K. M. Newell (1993) to movement, and (b) the movement spatial and temporal outcome variability are complementary and well predicted by an equation treating the variance of force and time in Newton's 2nd law as independent random variables. Collectively, the findings suggest that movement outcome variability is the product of a coherent space–time function that is driven by the nonlinear scaling of the force–time properties of the initial impulse.

Key words: movement speed–accuracy, spatial–temporal error, timing task

Interest in human movement variability has focused on the identification of the source of movement error and the explanation of the speed–accuracy error function. The accepted notion of the speed–accuracy tradeoff holds that more rapid movements lead to increased error; therefore, movement speed has to be slower if greater accuracy and less variability is required for a given task. Many examples of the speed–accuracy tradeoff are evident in a variety of tasks requiring spatial accuracy (e.g., sports, industry). However, there seems to be an opposite speed–accuracy relation in timing tasks, in that variable timing error decreases systematically as movement speed increases (Newell, Carlton, Carlton, & Halbert, 1980; Newell, Hoshizaki, Carlton, & Halbert, 1979). That apparent paradox in terms of the influence of movement speed on movement spatial and temporal accuracy has proved a challenging problem in understanding how a task should be performed optimally when both space and time are task criteria (Hancock & Newell, 1985; Newell, 1980; Newell, Carlton, Kim, & Chung, 1993).

Although there have been a number of attempts to explain movement accuracy, there is no common agreement about the nature of the speed–accuracy function in either spatial or temporal error. The speed–accuracy relations that have been proposed include the following: (a) a logarithmic speed–accuracy tradeoff (Crossman & Goodeve, 1983; Fitts, 1954); (b) a linear speed–accuracy tradeoff (Meyer, Smith, & Wright, 1982; Schmidt, Zelaznik, Hawkins, Frank, & Quinn, 1979); (c) a logarithmic timing error function as speed increases (Newell et al., 1980); and (d) a square function of speed and accuracy (Meyer, Abrams, Kornblum, Wright, & Smith, 1988). The diverse descriptions of the movement speed–accuracy relation have made it difficult for theorists to produce a general model of the speed and accuracy function (but, see Meyer et al., 1988; Plamondon & Alimi, 1997).

The different functions for the speed–accuracy tradeoff may arise in part from the different tasks that have been used in examining movement accuracy (e.g., Wright & Meyer, 1983; Zelaznik, Mone, McCabe, & Thaman, 1988). The incongruity among the extant movement accuracy functions might also arise from range effects of the diverse movement conditions examined in experimental work. For example, the differences between the linear and the nonlinear speed–accuracy tradeoff accounts may have resulted

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from the different range of movement speed conditions employed in the respective experiments. That is, the linear speed–accuracy tradeoff that is approximated well over limited spatial–temporal ranges of motion might turn out to be more obviously nonlinear over the full range of movement space–time parameter values (Hancock & Newell, 1985).

Schmidt et al. (1979) proposed that the variability in movement error is directly related to the variability in the associated impulse. In other words, the kinematic variability of movement outcome is directly related to the kinetic variability of the movement dynamics. One hypothesis of the impulse variability theory is that spatial error in an aiming task is directly proportional to impulse. In contrast, timing error has been proposed to be uninfluenced by increments of movement amplitude and, hence, average velocity within a given movement time. The results of subsequent experimental studies have shown that those predictions cannot be generalized and that they appear to be approximated only over limited portions of the potential space–time window of motion for a given effector system (Newell et al., 1980; Newell et al., 1993; Newell et al., 1979).

A significant contribution of the Schmidt et al. (1979) impulse variability theory for movement accuracy is that it linked the kinematics of movement error to the kinetics of motion; but the relation between the variability of movement impulse and the associated variability of movement error has still not been established, in part because there have been no direct measures of impulse in the extant experimental work on movement speed–accuracy relations. Another fundamental problem is that there are several features of the impulse that tend to covary as the size and the shape of the initial force–time trace change (Newell & Carlton, 1985, 1988). For example, increasing peak force while holding the duration of the impulse constant also leads to covariation in initial rate of the change of force in the impulse. Similarly, increasing force duration while holding peak force constant also changes the initial rate of force production. Thus, the effect of movement impulse on movement accuracy requires one to independently manipulate at least peak force, rate of force production, and force duration in order to realize the relation between impulse and movement error.

Our central focus in the current study was to examine the relation between movement impulse properties in determining movement variability in a timing task where the outcome of movement time is essentially determined by the initial impulse of motion (see Carlton & Newell, 1988). In particular, we wanted to test the degree to which the force variability scaling equations developed for isometric tasks by Carlton et al. (1993) can predict the spatial–temporal accuracy of a movement task. Furthermore, we extended the development of those scaling equations to derive a similar function for force duration variability. We also wanted to examine the degree to which Newton's second law of motion can predict movement variability, given the assumption that the variance in force and time reflects that of independent random variables. In those predictions, shape constancy of the impulse is assumed, which will not be strictly held in the movement tasks, but that working assumption is a first step to developing the appropriate equations and testing the notion that the variance of force and time reflects that of independent random variables. Examining movement accuracy from only the initial impulse of the movement allowed us to link those two scaling approaches together, because force duration was set as the movement time of the task.

Although tasks tend to be defined by properties of the task criterion—for example, timing tasks are presumably called such because the duration of the movement is the primary or only dependent variable—it is the case that all movement takes place in both space and time. Indeed, temporal properties of movement are determined at a particular place, and spatial properties of movement are determined at a particular time. Thus, the spatial and temporal properties of movement can be characterized in a coherent space–time framework (Hancock & Newell, 1985) so that timing error and spatial error are related when measured with respect to each other in a coherent frame of reference. Typically, in laboratory movement tasks the determination of the temporal and spatial criteria is arbitrary, according to the practical demands or the measurement traditions of a given task. No consideration is given to the space–time movement concept, even if spatial error and timing error are actually measured simultaneously. Our second focus in the current experiments was to examine movement accuracy within the space–time predictions of Hancock and Newell (1985) and to link the scaling properties of the impulse to the space–time movement error framework. In some recent experiments, movement error data have been produced that show the complementary nature of movement spatial and temporal error (Newell, Carlton, & Kim, 1994; Newell et al., 1993), but in those experiments, the properties of the initial impulse were not linked to the error functions.

In summary, in the three experiments reported here we examined the relation between the initial impulse and the space–time properties of movement variability. The key question of the study was whether the nonlinear impulse variability functions established in isometric tasks (Carlton et al., 1993; Newell & Carlton, 1988) also predict movement accuracy in space–time. A so-called timing task was used in which subjects were required to move a single limb (forearm) through a range of motion in a particular movement time. A set of task conditions with a range of parameter values of movement amplitude and time was examined. By determining the error from the spatial criterion at the criterion time, we also measured spatial error (see also Newell et al., 1993).

EXPERIMENT 1

Our purpose in this experiment was to examine the effect of scaling force with a constant force duration on movement space–time outcome variability. We obtained the force scaling experimentally by changing, over conditions, the range of motion while maintaining a constant movement time. Those experimental conditions should reveal how scaling the size of impulse contributes to the variability of spatial and temporal error.

Method

Participants

The participants were 6 volunteers from the University of Illinois at Urbana–Champaign. All of the participants were right-handed. Their mean age was 31 years (range = 27 to 37 years).

Apparatus

Participants placed their dominant arm on an elbow angular displacement bar. The bar could rotate about the horizontal plane. The bar was made of an L-shaped steel rod with a diameter of 1.91 cm. The long segment of the bar (65 cm) was vertically attached to the front face of a standard-height table by two pillow blocks, which allowed the bar to rotate freely about its vertical axis. The short segment of the bar (45 cm) rotated in the horizontal plane 18 cm above the height of the tabletop. We attached an arm support to the short horizontal segment to help the participant rest his or her forearm. A 9-cm-long cylindrical-shaped handle was attached to the distal end of the bar and was used by participants to grasp the bar during the elbow flexion movement. We varied the circumference of the handle from 9 to 12 cm so that it fit the shape of the participant's hand comfortably. The handle was 1 cm above and perpendicular to the short segment of the bar. The handle could be moved along the length of the bar so that subjects with different arm lengths could be accommodated. We mounted a one-turn 10-kΩ potentiometer (Allen Bradley) at the base of the long vertical segment of the rod to record displacement, and we attached an accelerometer (Entran Devices, 10 g) to the distal end of the horizontal segment of the bar at a distance of 40 cm from the axis of rotation to measure tangential acceleration.

A standard straight-backed chair of normal height was positioned adjacent to the arm bar. The chair was facing away from the table so that the axis of rotation of the bar was directly under the right elbow joint of the seated participant. Two markers were used for the target amplitude and the final stop position. Both markers were 25-cm-long, 8-mm-diameter dowel rods. The markers for the target and stop position projected vertically from a movable table. The markers were set just beyond the radius of the horizontal section of the arm bar and could be positioned to mark any desired target amplitude and stop position. A red warning light and green start light were placed just beyond the target marker and approximately 90 cm from the participant. A protractor and pointer were attached to a platform about the axis of rotation of the bar and allowed the experimenter to set the target angle and stop position.

The accelerometer, potentiometer, and lights were interfaced with a Hewlett Packard PDP1 1/73 DEC LAB computer. The acceleration and displacement data were sampled at a rate of 1000 Hz. After amplification, the acceleration and potentiometer signals were passed through a 10- and a 35-Hz low-pass filter, respectively.

Procedure

The participant sat in the chair, with the right arm on the arm rest and the hand grasping the handle. By adjusting the handle position, we set the right elbow directly over the axis of rotation of the arm bar. While waiting for the movement start, the lower arm of the participant was parallel to the ground with flexion of 30° from full extension, and the upper arm was parallel to the ground and supported laterally to the side. We placed a mechanical stop on the apparatus at an elbow angle of 30° to collect data from a constant starting position. The participant was required to perform elbow flexion from the start position, to pass through the designated target, and to stop at the designated position for a given condition. We adjusted the stop position for each condition to constrain the positive acceleration area of the acceleration-time signal to the designated range of motion. That manipulation creates a situation in which the movement time is determined essentially by the initial impulse (Carlton & Newell, 1988).

The warning and start lights were controlled by the computer and were located near the target marker. We randomly used four different foreperiods (750, 1,000, 1,250, and 1,500 ms) between the warning and start lights to prevent anticipation. The participant was instructed that this was not a reaction time experiment in which one has to react as rapidly as possible to the start signal. The participant initiated the movement when ready and attempted to move through the criterion range of motion in a time as close as possible to the criterion movement time and to stop at the stop position. After completing each movement, the participant returned the horizontal bar to the start position and received knowledge of results of the movement time for that trial. The intertrial interval was about 8 s.

Experimental Design

Each participant completed five range of motion–time conditions. The movement time was 150 ms, and the ranges of motion were 5, 10, 20, 30, and 40°. Each condition consisted of 60 trials, and the last 40 trials were used for data analysis. Testing was conducted over three testing sessions within a 1-week period. The order of presentation of the experimental conditions was randomized for each participant.

Results

In Experiment 1, we examined the relation between impulse variability and movement space–time variability by changing the range of motion over a constant criterion movement time. In Figure la, exemplar trials are presented, showing how acceleration changed over time in this task for

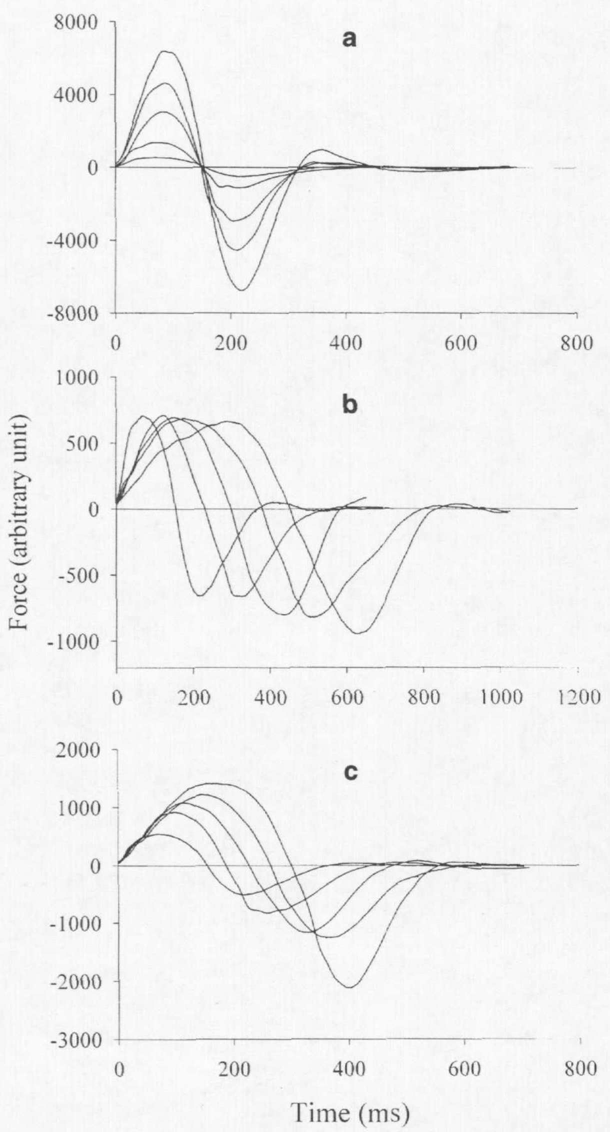

FIGURE 1. An exemplar trial from each of the five movement conditions that demonstrates (**a**) the relation of force parameter scaling while force duration is held constant (Experiment 1), (**b**) the relation of time parameter scaling while peak force is held constant (Experiment 2), and (**c**) the relation of time and force parameter scaling so that the rate of initial force production of the impulse is kept constant (Experiment 3).

the five range of motion conditions. It is interesting to note that the time to peak force increased slightly with increasing peak force: With force duration held constant, the impulse did not scale proportionally in force, as has been postulated in models of single-limb motion (e.g., Schmidt et al., 1979).

The mean, standard deviation, and coefficient of variation for a number of movement parameters as a function of the amplitude conditions are depicted in Table 1. *F* ratios based on within-participant one-way analyses of variance (ANOVAs) over the five ranges of motion for the movement-dependent variables are also shown in Table 1. The within-participant standard deviation was taken as the respective measure of variability in the different kinematic and kinetic variables.

Impulse Properties

Force and impulse were inferred from acceleration: Because the mass of the system was constant, the acceleration was proportional to force, and the area under the positive acceleration-time curve was proportional to initial impulse. The force unit, therefore, was arbitrary in this and the subsequent experiments. In the reported experiments, tangential acceleration (a_t) was measured. The a_t is the product of angular acceleration and the radius of the movement or the distance between the axis of rotation and the accelerometer ($a_t = r\alpha$). Because the radius was constant in the experiments, the tangential acceleration was proportional to angular acceleration ($\alpha \approx a_t$). The moment or torque (T) is a product of the angular acceleration and the moment of inertia of the system, that is, the forearm and arm bar assembly ($T = I\alpha$). Because the moment of inertia was constant for a participant, the moment was proportional to the angular acceleration ($T \propto \alpha$). The moment can also be described as a product of the force applied to the handle and the moment arm ($T = Fd$), where F is force and d is the moment arm. With the assumption that the moment arm was relatively constant in the experiments because of the relatively small ranges of motion and intermediate joint angles used, $T \propto F$, and, finally, tangential acceleration was proportional to force ($a_t \propto F$). However, to be consistent with terminology used in the movement variability literature, we report spatial parameters in degrees, temporal variability in milliseconds, and the kinetic variables in force and impulse throughout the article.

How the variability of peak force increased as a function of the force–time scaling equation developed for isometric force production in Carlton et al. (1993) is depicted in Figure 2a. Peak force variability generally tended to increase with average peak force and to decrease with increasing time to peak force. Although the force duration has previously been treated as a constant (e.g., Carlton et al., 1993; Schmidt et al., 1979), the observed force duration appeared to show a similar but inverse trend as the peak force variability, that is, the force duration variability increased with average force duration and decreased with increasing peak force. Therefore, a similar descriptive function was made for the force duration variability (see Appendix A). The variability of initial force duration as a function of peak force and the initial force duration is shown in Figure 2b.

Peak force variability increased at a negatively accelerating rate with increments of peak force, $F(4, 20) = 9.90$, $p < .01$. The coefficient of variation of peak force decreased at a negatively accelerating rate as peak force increased, $F(4, 20) = 18.15$, $p < .01$. The systematic departure from lineari-

TABLE 1
Means, Standard Deviations, and Coefficients of Variation (*CVs*) for Movement Variables in Experiment 1

	Range of motion/Movement time condition					
Statistic	50/150 ms	100/150 ms	200/150 ms	300/150 ms	400/150 ms	*F*(4, 20)
			Peak force (arbitrary units)			
M	830	1,919	3,778	5,068	6,463	
SD	166.5	320.0	478.0	536.0	537.8	9.90**
CV	.20	.16	.13	.10	.08	18.15**
			Force duration (ms)			
M	159.9	152.9	151.5	148.4	153.5	
SD	16.47	12.75	10.98	9.22	7.28	14.23**
CV	.10	.08	.07	.06	.05	14.04**
			Impulse (arbitrary units)			
M	73.3	162.2	315.3	458.4	618.2	
SD	7.70	14.65	20.65	22.73	30.53	17.96**
CV	.11	.09	.07	.05	.05	16.75**
			Range of motion (in degrees)			
M	4.92	10.07	19.78	29.65	39.73	
SD	.50	1.25	1.90	2.45	2.68	15.98**
CV	.11	.12	.10	.08	.07	7.05**
			Movement time (ms)			
M	149.09	147.37	150.68	151.13	151.77	
SD	13.89	12.03	10.58	8.65	6.65	17.66**
CV	.09	.08	.07	.06	.04	18.83**

**$p < .01$.

ty in the coefficient of variation confirms that peak force is not a sufficient variable to predict the peak force variability. The force duration variability systematically decreased from 16.47 to 7.28 ms as the peak force increased, even though the initial force duration and the shape of the initial impulse were similar across conditions. The analysis of variance confirmed a significant movement amplitude or peak force effect on the force duration variability, $F(4, 20) = 14.23$, $p < .01$.

As can be seen in Figure 2c, impulse variability was related to the size of impulse. That finding is consistent with the prediction of the Schmidt et al. (1979) model and provides evidence that the absolute level of impulse is an important variable in predicting impulse variability when the shape of impulse stays essentially unchanged across levels. However, the data of the present experiment do not follow the assumption that peak force variability is linearly related to peak force or that force duration variability is uninfluenced by the size of the impulse. Indeed, the coefficient of variation of impulse systematically decreased at a negatively accelerating rate as impulse increased, $F(4, 20) = 16.75$, $p < .01$. Under the assumptions that impulse is determined by force and time and that force and time are two independent random variables, one can use the component force and time properties to describe variability on the basis of the peak force variability function and the force duration variability function impulse (see Appendix B for a detailed derivation). A plot of the variability of impulse as a function of the combined scaling terms for peak force and duration of the impulse is shown in Figure 2d. A linear regression analysis on the data depicted in Figures 2c and 3d yielded R^2s of .96 and .98, respectively, indicating that scaling term organized the data about as well as impulse for this experiment.

Movement Variability

In Figure 2c, variable spatial error as a function of average movement velocity is illustrated. Variable spatial error increased at a negatively accelerating rate as average veloc-

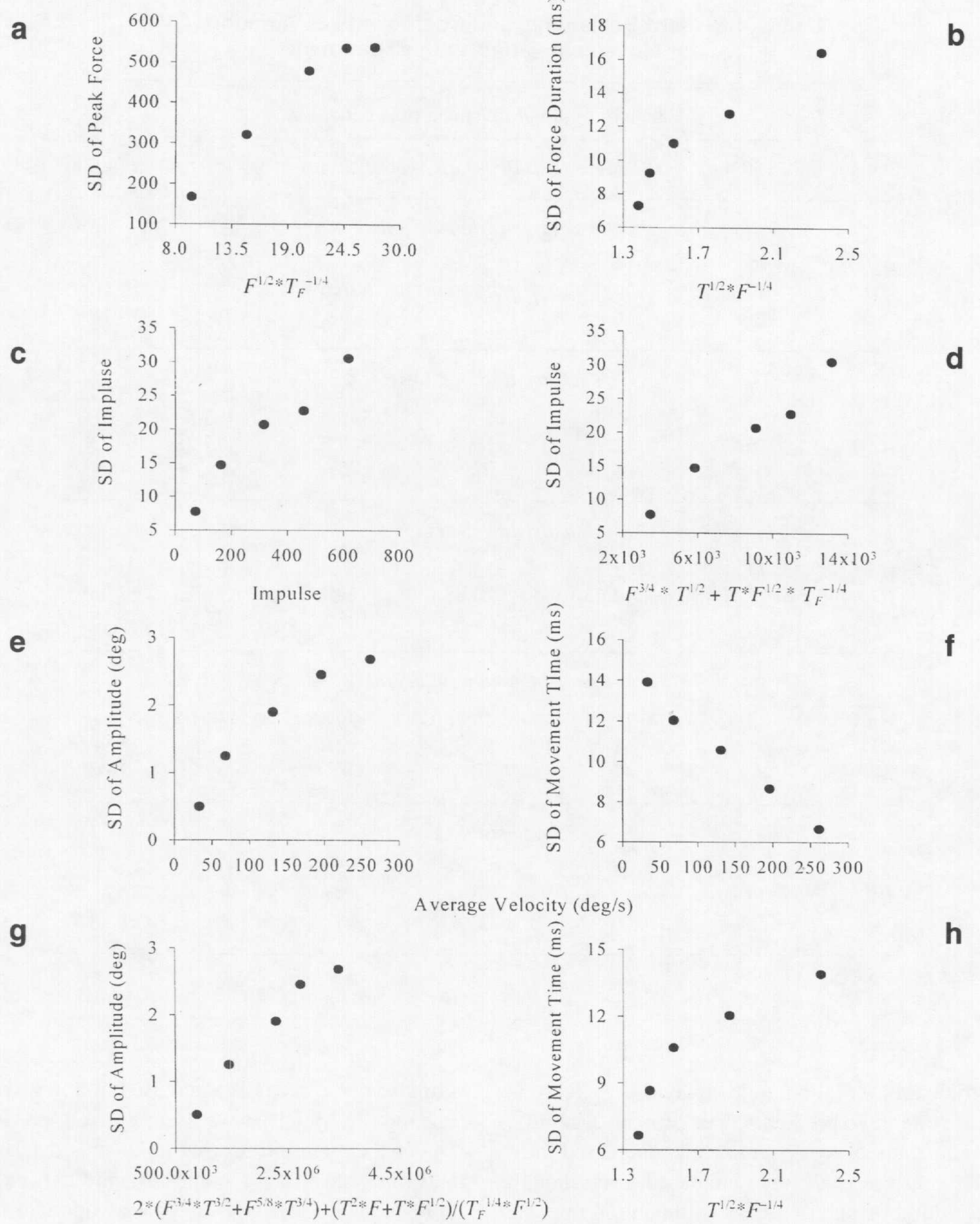

FIGURE 2. Experiment 1. The relation (**a–d**) between the variability of several impulse parameters and impulse scaling functions and (**e–f**) between the variability of movement outcome and average velocity, and (**g–h**) scaling functions derived from Newton's second law. See Appendices for an explanation of the terms shown on the *x*-axis.

ity increased, $F(4, 20) = 15.98, p < .05$. In Figure 2f, one can see how the variable timing error decreased significantly as the velocity increased, $F(4, 20) = 17.66, p < .01$. The paradox of the movement speed–accuracy tradeoff in space–time is revealed by the contrast between the spatial error data in Figure 2e and the timing error data in Figure 2f.

Impulse and Movement Variability

In examinations of force variability in isometric tasks, we have previously shown that the variability of impulse parameters follows a dimensional scaling function of force and time (Carlton & Newell, 1993; Carlton et al., 1993; Newell & Carlton, 1988). Here, we extended that relationship to the movement domain and examined movement accuracy in relation to the initial impulse. To examine the scaling relation between impulse and movement error in space, we used Newton's second law of motion with the additional assumption that force and time are independent random variables. The derivation of that scaling term, which is used in Figure 2g, is shown in Appendix C. We specifically designed the experimental tasks to align the movement time with the force duration, letting the description of the movement time variability be essentially the same as that of the force duration variability. In a more general situation in which the movement time is not the same as the force duration, however, one needs to specify the relation between the movement time and force duration in order to predict the kinematic property from the kinetic information. In Figures 2g and 2h, the variability of both spatial and temporal error in relation to this force–time scaling function are shown. The function seemed to fit the spatial error data better than the temporal error data; but in the present study it was the spatial dimension that was manipulated, and therefore had a greater range of parameter values than the movement time, which was essentially the same for each condition.

EXPERIMENT 2

In Experiment 2, we examined the influence on movement outcome variability of varying the time properties of the initial impulse while holding peak force constant. By changing both range of motion and movement time of the task, we manipulated the time scaling. We designed this experiment to identify how scaling the time properties of the same peak force of an impulse determines the variability of space–time movement error.

Method

Participants

The participants were 6 volunteers from the University of Illinois at Urbana–Champaign. All of the participants were right-handed. The mean age was 29.8 years (range = 27 to 32 years). None of the participants had taken part in Experiment 1.

Apparatus

The apparatus was identical to that used in Experiment 1. Participants produced a discrete movement in a given movement time and range of motion.

Procedures

The procedures were the same as those of Experiment 1, except for the different movement time and range of motion task constraints. Exemplar trials of how we obtained scaling of the duration with the same peak force of impulse by manipulating range of motion and movement time are shown in Figure 1b.

Experimental Design

There were five conditions, with movement times of 150, 225, 300, 375, and 450 ms and with ranges of motion of 5, 11.3, 20, 31.3, and 45°, respectively. The types of analyses used were the same as those conducted in Experiment 1.

Results

The means, standard deviations, and coefficients of variation for a number of impulse and movement outcome parameters are shown in Table 2. The F ratio from the one-way ANOVA for each variable over the range of motion and movement time combinations is also provided.

Impulse Properties

In Figures 3a and b are plots of peak force variability and force duration variability, respectively, as a function of the force–time scaling equations from Carlton et al. (1993) and Equation A2 (see Appendix A). The variability of those impulse parameters increased as the force–time scaling term increased. The range of values in the two force–time scaling terms was low in this experiment because of the particular movement conditions used. In Figures 3c and 3d are depicted the variability of impulse as a function of impulse magnitude and the scaling equation B7 (see Appendix B), with force and time as independent variables. Impulse variability increased similarly for both scaling terms. As is shown in Table 2, there was a reduction in peak force variability with increased time to peak force, but that change was not significant, $F(4, 20) = 2.13$, $p > .05$. The coefficient of variation of peak force variability decreased significantly as range of motion and movement time increased, $F(4, 20) = 3.35$, $p < .05$. Force duration variability increased essentially linearly with force duration. However, there was an interaction between peak force and force duration on force duration variability. Although the peak force variability across conditions was not significantly different, it systematically decreased as the force duration increased. That result suggests that the force and time properties of the impulse are dependent upon each other in predicting the variability function of the force and time properties of impulse. Impulse variability was highly correlated with impulse. The coefficient of variation of impulse was not significantly different across conditions, $F(4, 20) = 1.09$, $p > .05$ (see Table 2). Impulse seemed to be a good predictor of impulse variability for time parameter scaling as well as for force parameter scaling, as was illustrated in Experiment 1.

Movement Variability

Variable spatial error and variable timing error, respectively, as functions of average movement velocity are shown in Figures 3e and 3f. The data trends shown in both of those figures indicate a high predictability of movement spatial and temporal error in terms of average velocity. The variable spatial error showed a trend similar to that of the variable timing error, and both error functions showed small but systematic departures from linearity. Indeed, the trend for spatial and temporal error appeared to be that of an ogival function.

The variable spatial error generally increased as the range

TABLE 2
Means, Standard Deviations, and Coefficients of Variation (*CVs*) for Movement Variables in Experiment 2

Statistic	Range of motion/Movement time condition					
	50/150 ms	100/150 ms	200/150 ms	300/150 ms	400/150 ms	$F(4, 20)$
	Peak force (arbitrary units)					
M	758	884	793	778	735	
SD	141.12	124.7	130.7	114.3	103.7	2.13
CV	.19	.14	.16	.15	.14	3.35*
	Force duration (ms)					
M	170.9	232.5	298.1	357.9	419.2	
SD	17.00	19.18	27.38	32.42	36.43	12.42**
CV	.10	.08	.09	.09	.08	1.27
	Impulse (arbitrary units)					
M	71.4	116.0	141.3	169.9	192.5	
SD	6.92	9.35	11.55	13.58	14.85	7.94**
CV	.10	.08	.08	.08	.08	1.09
	Range of motion (in degrees)					
M	5.05	11.44	20.15	31.90	45.42	
SD	.61	1.28	2.58	3.41	3.81	31.43**
CV	.12	.11	.13	.11	.08	3.13*
	Movement time (ms)					
M	150.6	224.6	297.8	372.1	447.8	
SD	14.58	15.73	21.71	25.42	25.58	10.62**
CV	.10	.07	.07	.07	.06	7.56**

$^*p < .05$, $^{**}p < .01$.

of motion and average velocity increased. The coefficient of variation of the range of motion decreased significantly with increases in average velocity, $F(4, 20) = 2.13$, $p < .05$. The variable timing error had a tendency to increase as the movement time increased, and the general trend suggested a nonlinear function. The coefficient of variation of movement time also decreased significantly with increases in average velocity, $F(4, 20) = 7.56$, $p < .01$.

Impulse and Movement Variability

In Figures 3g and 3h are shown, respectively, the variability of movement amplitude and movement time as a function of the derived scaling equations shown in Appendix C. Again, there was a similar shaped function for the variability of movement in both space and time, and it appeared to be ogival.

EXPERIMENT 3

We designed Experiment 3 to examine the effect of initial impulse on movement outcome variability by varying both peak force and time values of the initial impulse. To keep the average rate of initial force generation essentially constant, we manipulated the force and time properties of the impulse by changing both the range of motion and the movement time of the task. The experimental conditions enabled us to examine how force and time function interactively in determining movement space–time error.

Method

Participants

The participants were 6 volunteers from the University of Illinois at Urbana–Champaign. All of the participants were right-handed. The mean age was 31 years (range = 24 to 34 years). None of the participants had taken part in the previous experiments.

Apparatus

The apparatus was identical to that used in Experiment 1. Participants again produced a discrete movement over a set of particular movement times and ranges of motion.

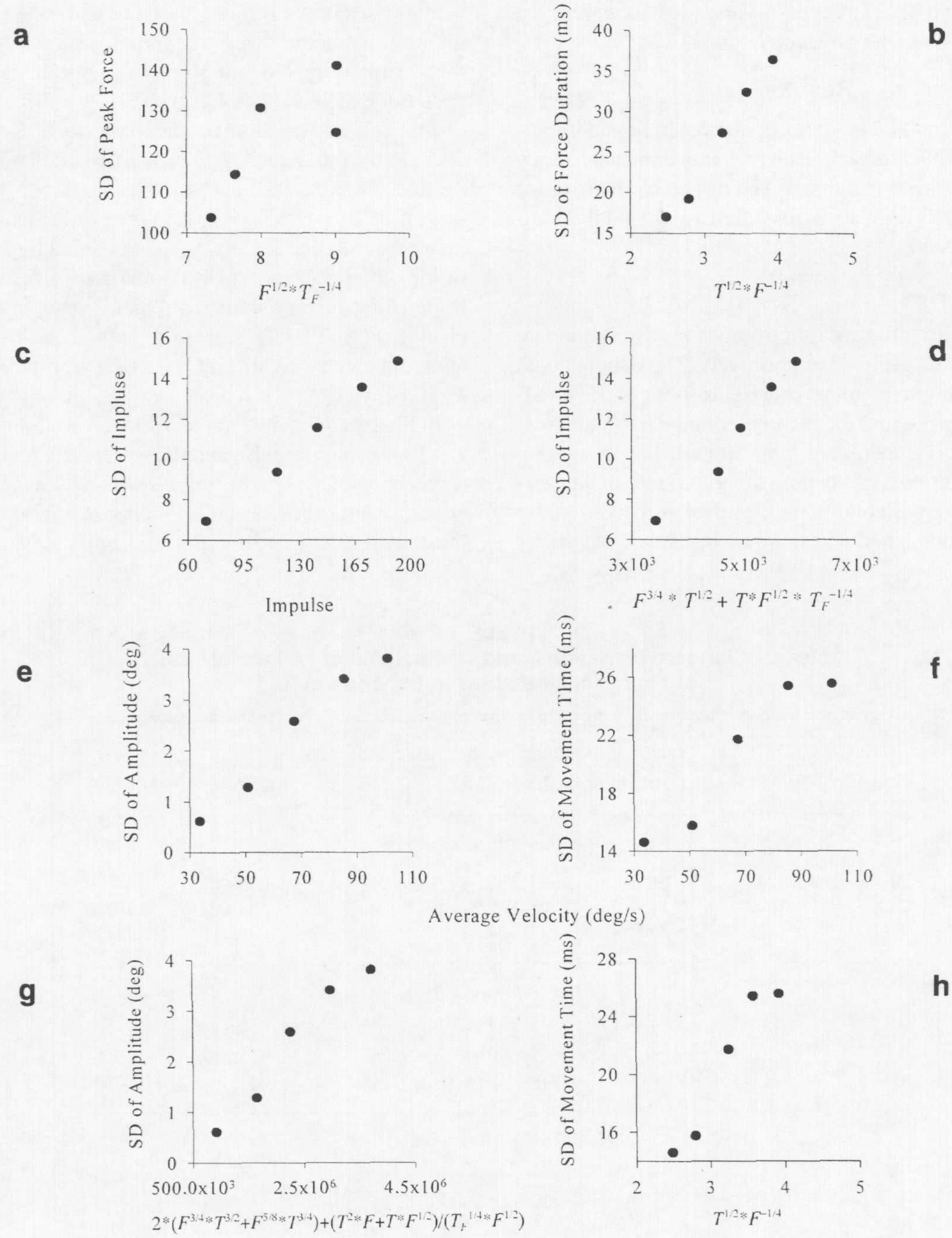

FIGURE 3. Experiment 2. The relation (**a–d**) between the variability of several impulse parameters and impulse scaling functions and (**e–f**) between the variability of movement outcome and average velocity, and (**g–h**) scaling functions derived from Newton's second law. See Appendices for an explanation of the terms shown on the *x*-axis.

Procedures

The procedures were the same as those used in Experiment 1, except for the different movement time and range of motion task constraints. How we obtained peak force and time scaling of the impulse with constant initial rate of force by manipulating range of motion and movement time is shown in Figure 1c.

Experimental Design

The conditions had movement times of 150, 188, 225, 263, and 300 ms, and the respective ranges of motion were 5, 10, 17, 27, and 40°. The types of analyses used were the same as those conducted in Experiment 1.

Results

The means, standard deviations, and coefficients of variation for a number of impulse and movement accuracy parameters are shown in Table 3. The respective *F* ratios for the one-way ANOVA tests across the experimental conditions are also listed.

Impulse Properties

The variability of peak force and force duration are shown in Figures 4a and 4b, respectively. The data for both of those variables were not as orderly as those of the previous experiments because the parameter range for those scaling terms in this experiment was limited; that limitation resulted from our goal of keeping the initial rate of acceleration the same across conditions. The peak force variability was not proportional to the peak force; instead, it was closely related to the initial rate of force production. That result is a good example of how the force and time properties of impulse interact to determine impulse variability. The peak force variability did not show any systematic change as peak force increased, $F(4, 20) = 0.10$, $p > .05$. However, the coefficient of variation of the peak force systematically decreased from .22 to .12 as peak force increased (see Table 3), and the differences across the conditions were significant, $F(4, 20) = 9.31$, $p < .01$. In the present experiment, the increment of peak force was accompanied by an increment in the duration of force by a similar ratio. The analysis of force duration variability did not show any significant change, $F(4, 20) = 2.79$, $p > .05$, but the coefficient of variation indicated a significant reduction as impulse increased, $F(4, 20) = 11.60$, $p < .05$.

In Figures 4c and 4d are shown the variability of impulse as a function of impulse and the force–time scaling relation, respectively. In general, impulse increased systematically across both of those scaling functions. When we examined the relation between impulse and impulse variability (see

TABLE 3
Means, Standard Deviations, and Coefficients of Variation (*CV*s) for Movement Variables in Experiment 3

	Range of motion/Movement time condition					
Statistic	50/150 ms	100/150 ms	200/150 ms	300/150 ms	400/150 ms	$F(4, 20)$
	Peak force (arbitrary units)					
M	774	1,040	1,226	1,353	1,493	
SD	180.0	168.7	179.2	182.2	174.8	0.10
CV	.22	.16	.14	.13	.12	9.31**
	Force duration (ms)					
M	169.3	192.9	227.1	257.7	299.1	
SD	20.78	17.65	19.15	22.95	21.95	2.79
CV	.13	.09	.08	.09	.07	11.60**
	Impulse (arbitrary units)					
M	67.7	115.7	168.8	213.0	271.6	
SD	7.52	10.17	12.10	14.38	17.40	25.98**
CV	.11	09	07	07	06	11.98**
	Range of motion (in degrees)					
M	5.05	9.97	16.93	27.34	39.46	
SD	.69	1.24	1.82	2.88	3.37	46.42**
CV	.13	.12	.11	.10	.08	3.43*
	Movement time (ms)					
M	151.0	188.7	226.3	262.0	302.8	
SD	16.31	15.16	14.99	17.19	15.97	0.53
CV	.11	.08	.07	.07	.05	8.42**

$^*p < .05$, $^{**}p < .01$.

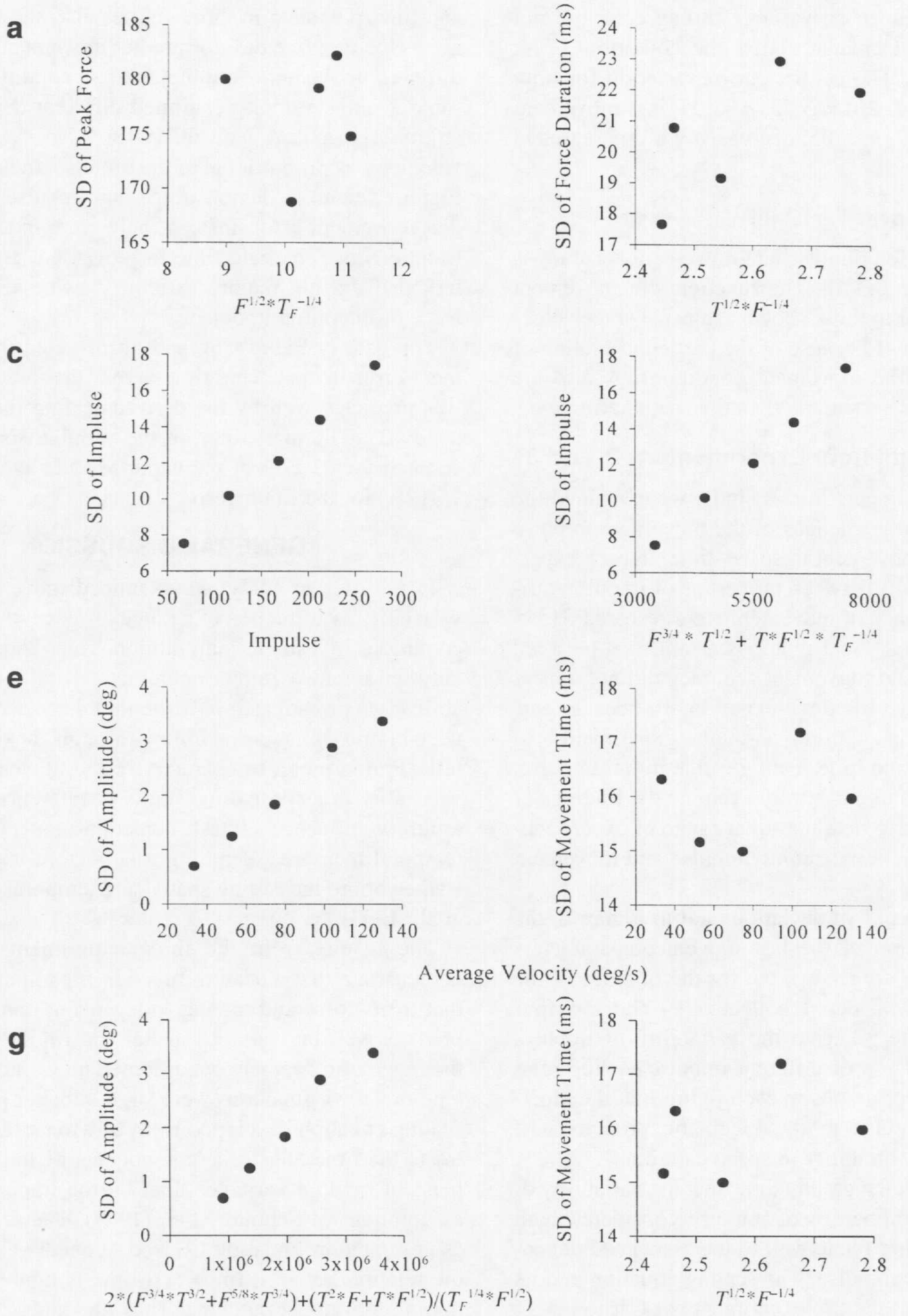

FIGURE 4. Experiment 3. The relation (**a–d**) between the variability of several impulse parameters and impulse scaling functions and (**e–f**) between the variability of movement outcome and average velocity, and (**g–h**) scaling functions derived from Newton's second law. See Appendices for an explanation of the terms shown on the *x*-axis.

Figure 4c) by using regression analysis, we found that impulse accounted for more than 99% of the variance in impulse variability. Similar regression values emerged from the use of the scaling term (Figure 4d).

Movement Accuracy

Variable spatial error and variable timing error, respectively, as a function of average movement velocity are shown in Figures 4e and 4f. The variability of spatial error increased over average velocity, $F(4, 20) = 46.42$, $p < .01$. The analysis of variance on variable timing error did not show any significant change across the conditions, $F(4, 20) = 0.53$, $p > .05$. The coefficient of variation for both range of motion, $F(4, 20) = 3.43$, $p < .05$, and movement time, $F(4, 20) = 8.42$, $p < .05$, decreased with increases in average velocity.

Impulse and Movement Variability

The relation between impulse and movement accuracy is shown in Figures 4g and 4h. The functions for movement variability are similar to those shown against average velocity in Figures 4e and 4f because of the particular scaling of the impulse across the movement conditions. Again, the timing error was less systematic than the spatial error.

Combined Results from Experiments 1, 2, and 3

In the three experiments in this study, we examined the influence of force–time changes of the movement impulse on the accuracy of movement in space–time. In each experiment, certain relations between properties of impulse variability and the accuracy of movement were revealed. However, some of the functional relations established between impulse variability and movement accuracy did not appear to be consistent across the three experiments, because certain task manipulations within a particular experiment produced limited force and time scaling of the initial impulse. Therefore, one needs to coherently examine the findings of all three experiments to obtain a fuller range of experimental conditions in which to examine impulse and movement accuracy.

In Figure 5, the standard deviation, and in Figure 6, the coefficient of variation of the key movement parameters examined previously are shown, but for the findings of the three experiments considered collectively. The top four graphs in each figure relate to the variability of impulse parameters as a function of different impulse scaling relations. The bottom four graphs in each figure link the kinetic properties of impulse to the kinematic properties of movement outcome variability in space and time.

Overall, the data in the graphs show that the variability of the force and time properties of impulse considered both separately and together relate well to the associated dependent variables of the impulse. The scaling equation and its adaptation from the isometric experiments of Carlton et al. (1993) predict in a movement task the peak force and force duration considered separately, as shown in Figures 5a and 5b. The variability of impulse was fitted very well ($R^2 = 97.91\%$) by the scaling function of Equation B7 (see Appendix B), which accounted for a slightly greater percentage of the variance than impulse alone ($R^2 = 96.25\%$). The intercept of the scaling function of Equation B7 was also close to zero.

The variability data for spatial and temporal error as a function of average velocity, shown in Figures 5e and 5f, confirmed previous findings (Newell et al., 1993) and the paradox of the speed accuracy relations in terms of space and time. The data in those figures also showed that average velocity alone did not predict movement error, in that different movement amplitude–time conditions with the same average velocity exhibited different degrees of variability (Hancock & Newell, 1985). Also, because average velocity was proportional to the impulse, the data shown in Figures 5e and 5f demonstrated that impulse does not predict movement error. Instead there were independent contributions of force and time to movement error, even in a task that did not require extreme skewness in the rate of force of the initial impulse.

The data in Figures 5g and 5h revealed that the movement error in space and time across the three experiments was predicted well by the derived scaling function of Appendix C. The regression on the variable spatial error accounted for 92.22% of the variance and slightly greater, at 95.12%, for the timing error variance.

GENERAL DISCUSSION

In the present study, we examined movement outcome variability as a function of a range of space–time movement conditions in a single-joint motion. To provide a direct link between impulse, impulse parameters, and movement variability, we investigated only the initial acceleration component of movement in a follow-through movement timing task. Furthermore, to characterize the movement outcome variability in a space–time frame of reference, without the arbitrary influence of task constraints on the spatial and temporal measures of hitting a target, we used a coherent framework to measuring spatial and temporal error (Newell et al., 1993).

The results from the three experiments collectively demonstrate that a scaling function for impulse variability that treats force and time as independent random variables predicts well the variable spatial and timing errors across the range of movement conditions. The variable spatial and temporal error functions were slightly better predicted by a scaling equation developed from Newton's second law that treats the time and force components of impulse as independent random variables than by the impulse variability assumptions of Schmidt et al. (1979). The scaling formula as presented in Equation C6 (see Appendix C) showed that the relation between force and time is nonlinear in determining movement outcome. Thus, the kinetic properties of the initial impulse can predict the kinematics of movement error as proposed by the general background assumptions of the impulse variability model of Schmidt et al. (1979), but the global measure of impulse is not itself sufficient to predict movement error, because it cannot accommodate the influence of the changing rate of force on movement outcome (Carlton & Newell, 1988). That conclusion is counter to the general notion of the linear speed–accuracy relation proposed in current impulse variability models (Meyer et. al., 1982; Schmidt et al., 1979) and suggests a number of factors in relation to the development of a general space–

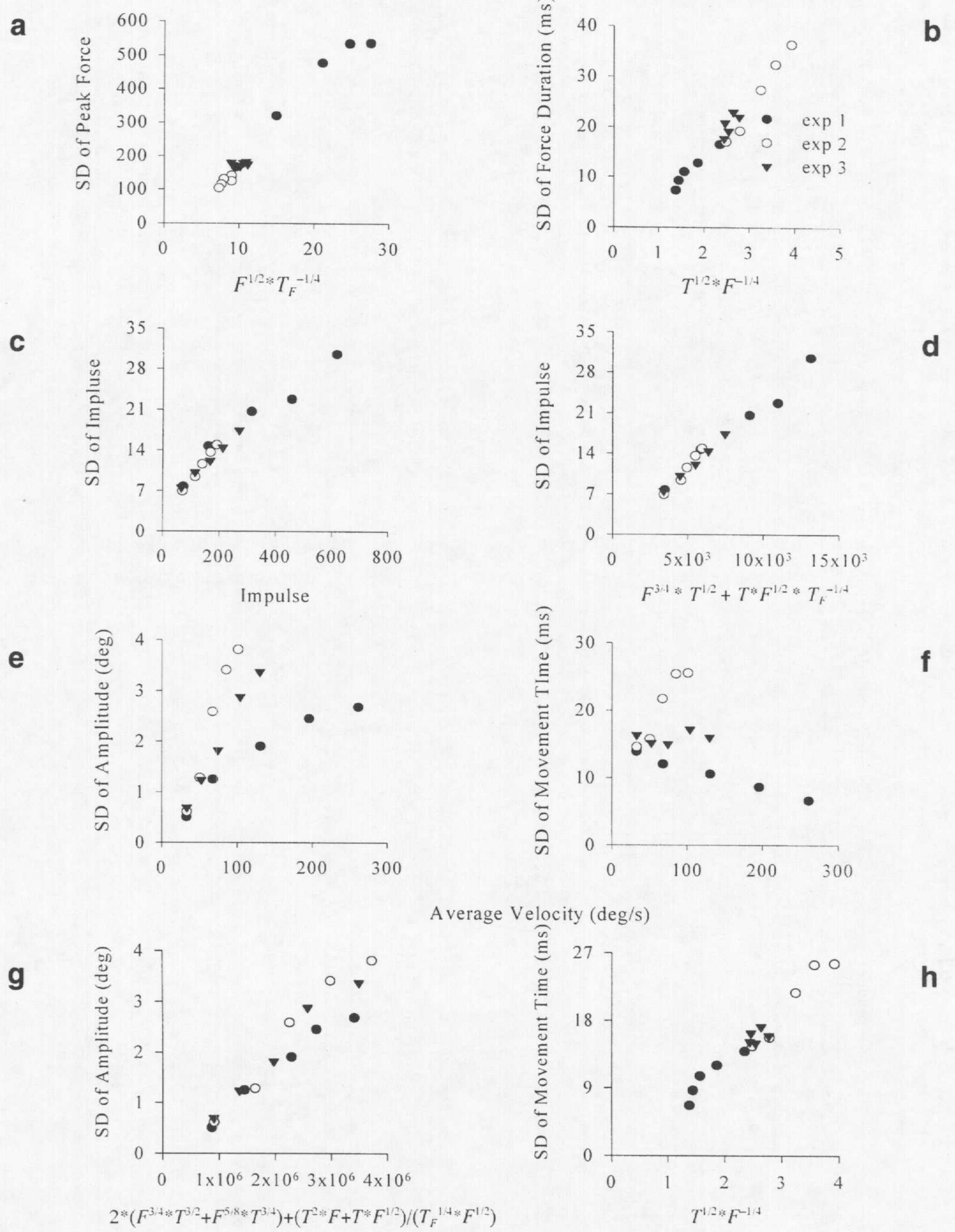

FIGURE 5. The synthesized relation over all experiments (**a–d**) between the variability of several impulse parameters and impulse scaling functions and (**e–f**) between the variability of movement outcome and average velocity, and (**g–h**) scaling functions derived from Newton's second law. See Appendices for an explanation of the terms shown on the *x*-axis.

time account of movement accuracy (Hancock & Newell, 1985). Aspects of the impulse variability model have been tested previously in both aiming tasks (Schmidt et al., 1979; Zelaznik et al., 1988) and timing tasks (Carlton & Newell, 1988; Newell et al., 1980; Sherwood, 1986), but the findings from the current study reveal more comprehensively the relation between impulse and movement error.

The amount of spatial and temporal error variance explained by the scaling equations developed here could probably be increased under different task conditions and

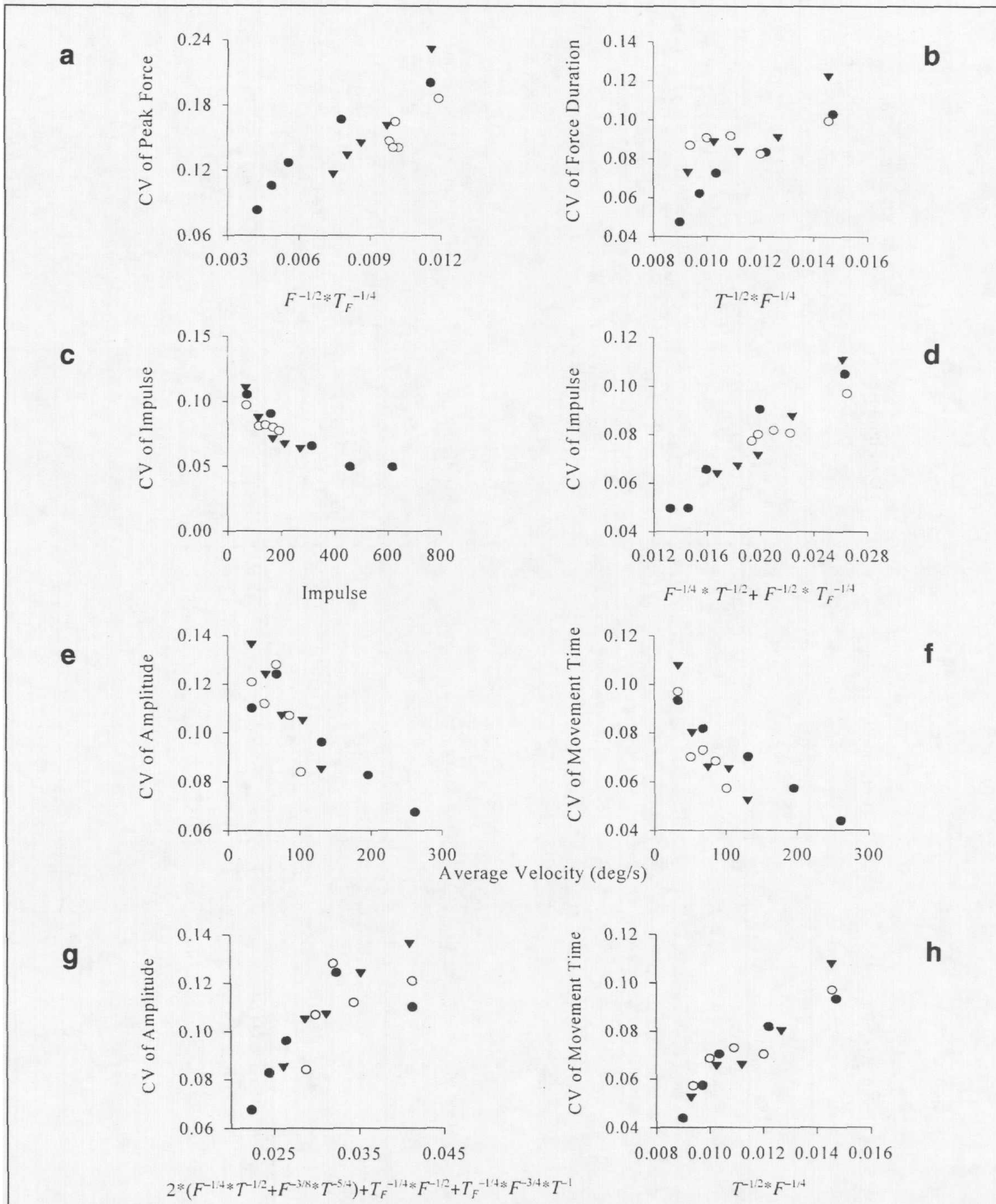

FIGURE 6. The synthesized relation over all experiments (**a–d**) between the coefficient of variation of several impulse parameters and impulse scaling functions and (**e–f**) between the coefficient of variation of movement outcome and average velocity, and (**g–h**) scaling functions derived from Newton's second law. See Appendices for an explanation of the terms shown on the x-axis.

measurement techniques. Although the participants generally held, on average, the force duration to that of the task movement time, there were small differences on most trials. That difference reduced by a small degree the amount of variance explained by the scaling equations. The impulses also showed small deviations from a symmetrical force–time shape, and those departures from a basic assumption of the equation modeling also reduced the variance accounted for in the relation between impulse and spatial and temporal error variability. Overall, though, with the equations used here we showed that modeling impulse and movement variability with the assumption of force and time as independent random variables approximates well the movement outcome. Indeed, the results obtained encourage the further development of equations that handle departures in the shape of the initial impulse, along the lines that we developed previously for isometric force output (Carlton et al., 1993). A more complete account for movement speed–accuracy issues would also include, of course, the force variability developed beyond the initial impulse in the spatial–temporal evolution of the movement, because that is the more common situation in movement accuracy tasks.

The results from the present study show that impulse itself (Schmidt et al., 1979) is not sufficient to explain the movement error variability in a timing task, even though the departures from a symmetrical impulse shape were small. The movement conditions in which we changed space by holding time constant, or vice versa, to account for the speed–accuracy relation lead to a limited explanation in linking the impulse variability to the movement error variability through the whole space–time range of movement. For example, the linear relation between impulse and movement amplitude proposed by Schmidt et al. (1979) is particularly weak in predicting error across the scaling of both the force and time of impulse.

The findings of the present study did not support the linear speed–accuracy tradeoff predicted from the symmetric scaling of force and time in impulse variability (Meyer et al., 1982). Whatever strategies were employed in changing the speed in space and time (e.g., force scaling), the average velocity accounted for the variable spatial error with a minimum R^2 of .935 in each experiment. However, when the speed control strategies were considered together, the average velocity explained variable spatial error with an R^2 of .294. Therefore, Meyer et al.'s (1982) prediction of the speed–accuracy relation appears limited to only a certain range of force and time properties of the impulse.

The current findings also build on the preliminary space–time error findings that have been previously reported (Carlton et al., 1993; Newell, 1980; Newell et al., 1993) and the general theorizing for a space–time account of movement accuracy (Hancock & Newell, 1985). In the present study, we found that any nonlinearity of variable spatial error predicted from average velocity covaried with changes in variable timing error. The systematic decrease of variable timing error with increased average velocity is evidence for the dependency between variable spatial error and variable timing error. The decreasing function of variable timing error with increments of average velocity is complementary to the negatively accelerating change of variable spatial error with increments of average velocity (Newell et al., 1993).

In summary, as shown in the present study, there is a strong relation between movement variability in space and time and the variability of the initial impulse. The approach taken here can be further developed to include asymmetrical impulses and movement conditions that have more than an initial impulse. Finally, complementary trends for spatial and temporal error were observed from the data, and the proposed space–time view of movement accuracy (Hancock & Newell, 1985)—that spatial error is traded for timing error in movement outcome—was confirmed.

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APPENDIX A

Development of Scaling Functions

For the development of the following scaling functions, we assumed that the force duration is impulse shape dependent. As a first attempt to predict the force duration variability, we assumed the straightforward situation in which a constant impulse shape is maintained across all the predicted conditions and the peak force occurs at the midpoint of the force–time curve. We used a general gaussian function to model the initial impulse shape in which the maximum force and the force duration can be manipulated independently. After observing the force duration variability data from the current experiments and from previous literature, we used the peak force variability function developed in Carlton et al. (1993) as the basis of the initial force duration variability function. The peak force variability function was proposed as

$$SD_F = K_1 F^{1/2}\, T_F^{-1/4}, \qquad \text{(A1)}$$

where SD_F is the standard deviation of peak force, K_1 is a constant, F is average peak force, and T_F is the average time to peak force. Thus, following the same trend, the initial force duration variability was proposed to have the following relation:

$$SD_D = K_2 T^{1/2}\, F^{-1/4}, \qquad \text{(A2)}$$

where SD_D is the standard deviation of the initial force duration, K_2 is a constant, T is the averaged initial force duration, and F is the averaged peak force. In all of our experimental conditions, the peak force generally occurred at the midpoint of the initial force duration; the initial force duration rather than the time to peak force was used as a parameter in the scaling function for the initial force duration variability. We did not take into account the ratio of time to peak force to initial force duration in the scaling function under the current experimental conditions. The formula captures the general idea that the initial force duration variability is determined by both peak force and the initial force duration and is positively related to the initial force duration and negatively related to peak force. The fractions in the exponents indicate the nonlinear relation between the variability of the initial force duration and the component force–time values and the relative contributions of those components to the variability. However, no effort has been made in terms of improving the goodness of fit. The specific exponent values were similar to the peak force variability function, indicating a general relation between the parameters rather than a specific fitting to particular values.

APPENDIX B

Derivation of the Impulse Variability Function

In deriving the impulse variability function, one assumes a symmetrical force–time curve with a scalable shape: Specifically, a Gaussian curve, $f(t) = h \cdot \exp[-1/2(t/a)]^2$, where h is a parameter determining the maximum height of the curve and a is another parameter determining the spread of the curve. First, assume that we have a constant unit peak force that occurs at the midpoint of the force–time curve. If we increase the duration of force production but maintain the peak force at the unit level, we must have a relatively flat force–time curve; if we maintain the duration constant while increasing the peak force, then we will have a short and sharp force–time curve. The impulse is the area under the force–time curve. We integrate the Gaussian curve from $-3a$ to $+3a$ to capture the start of initial impulse at about 0 unit force and the end of initial impulse at about 0 unit force level:

$$\begin{aligned} I &= \sum_{-3a}^{3a} h \cdot e^{-\frac{1}{2}\left(\frac{t}{a}\right)^2} dt, \\ &= h \cdot a\sqrt{2\pi} \cdot Erf\left[\frac{3}{\sqrt{2}}\right], \\ &= h \cdot a \cdot K_3, \end{aligned} \qquad \text{(B1)}$$

where the error function $Erf(3/\sqrt{2}) \approx 0.4822$. Impulse is proportional to the product of the maximum height and the spread (duration) of the curve. Assume the peak force and the force duration are two independent random variables; then the variance of the product of the two independent random variables can be determined from the following derivation.

If x and y are independent random variables, σ_x^2 is the variance of the random variable x, and σ_y^2 is the variance of the random variable y, and X and Y are the means of the random variables x and y, respectively, then σ_{xy}^2, the variance of the product of the two random variables x and y, can be calculated as the following:

$$\sigma_{xy}^2 = E[x^2y^2] - [E(xy)]^2, \qquad \text{(B2)}$$

$$\begin{aligned} E(x^2y^2) &= [\Sigma(X + \delta x)^2 (Y + \delta y)^2]/n, \\ &= X^2Y^2 + X^2\sigma_y^2 + Y^2\sigma_x^2 + \sigma_x^2\sigma_y^2, \end{aligned} \qquad \text{(B3)}$$

where δx and δy are the differences between individual scores and the sample means for variables x and y, respectively.

$$[E(xy)]^2 = (XY)^2 = X^2Y^2, \qquad \text{(B4)}$$

$$\begin{aligned} \sigma_{xy}^2 &= X^2Y^2 + X^2\,\sigma_y^2 + Y^2\,\sigma_x^2 + \sigma_x^2\,\sigma_y^2 - X^2Y^2, \\ &= X^2\,\sigma_y^2 + Y^2\,\sigma_x^2 + \sigma_x^2\,\sigma_y^2. \end{aligned} \qquad \text{(B5)}$$

If σ^2s are small in comparison with X^2 and Y^2, the last term can be ignored. Therefore,

$$\sigma_{xy}^2 = X^2\, \sigma_y^2 + Y^2\, \sigma_x^2. \qquad \text{(B6)}$$

By substituting X by F, σ_x by $K_1F^{1/2}\, TF^{-1/4}$, Y by T, and σ_y by $K_2T^{1/2}\, F^{-1/4}$ in Equation B6, we approximate

$$SD_{\mathrm{IMP}} \approx K_4\, (F^{3/4}T^{1/2} + TF^{1/2}\, TF^{-1/4}), \qquad \text{(B7)}$$

where SD_{IMP} is the standard deviation of initial impulse.

APPENDIX C

Derivation of SD_{IMP}

From Newton's second law of motion,

$$D = Kft^2/M, \qquad \text{(C1)}$$

where D is the distance traveled, K is a constant, f is the force applied for the movement, t is the movement time, and M is the limb mass (Meyer et al., 1982; Schmidt et al., 1979; Sears, 1958). Assume the mass of the limb is also a constant; therefore, the distance traveled is proportional to the product of the applied force and the square of the movement time. Let us assume again that force and time are two independent random variables, y and x, using the following derivation to determine the variance of the product yx^2:

To express the variance of x^2 in terms of $\sigma^{[x]2}$ and X, we have

$$\begin{aligned}\sigma_{x^2}^{\ 2} &= E\left[\left(x^2\right)^2\right]-\left[E\left(x^2\right)\right]^2,\\ &= E(x^4) - \{[E(x)]^2 + \sigma_x^2\}^2,\\ &= E(x^4) - (x^4 + 2\,X^2\sigma_x^2 + \sigma_x^4),\end{aligned} \qquad \text{(C2)}$$

and

$$\begin{aligned}E(x^4) &= [\Sigma(X + \delta x)^4]/n,\\ &= X^4 + 6\,X^2\sigma_x^2 + 4\,X\sigma_x^3 + \sigma_x^4.\end{aligned} \qquad \text{(C3)}$$

Therefore,

$$\begin{aligned}\sigma_{x^2}^{\ 2} &= X^4 + 6X^2\sigma_x^{\ 2} + 4X\sigma_x^{\ 3} + \sigma_x^{\ 4}\\ &\quad - (X^4 + 2X^2\sigma_x^{\ 2} + \sigma_x^{\ 4})\\ &= 4\,X^2\sigma_x^2 + 4\,X\sigma_x^3\\ &= 4\,X\sigma_x^2\,(X + \sigma_x).\end{aligned} \qquad \text{(C4)}$$

Next, to express the variance of the product of x^2 and y in terms of σ_x^2 and σ_y^2, we have

$$\begin{aligned}\sigma_{x^2y}^{\ 2} &= Y^2\sigma_{x^2}^{\ 2} + \overline{x^2}^{\,2}\sigma_y^2 + \sigma_{x^2}^{\ 2}\sigma_y^2,\\ &= Y^2\,[4X\,\sigma_x^2\,(X + \sigma_x)] + (X^2 + \sigma_x^2)^2\,\sigma_y^2\\ &\quad + [4X\,\sigma_x^2(X + \sigma_x)]\,\sigma_y^2,\end{aligned} \qquad \text{(C5)}$$

where $\overline{x^2}$ is the mean of the random variable x^2.

Here, we used peak force for the applied force and the initial force duration as the movement time because we designed the experimental tasks to align the movement time to the end of the initial force duration. By substituting X by F, σ_x by $K_1F^{1/2}\, TF^{-1/4}$, Y by T, and σ_y by $K^2T^{1/2}F^{-1/4}$ in Equation C5, we approximate

$$\begin{aligned}SD_{\mathrm{AMP}} &\approx 2(F^{3/4}T^{3/2} + F^{5/8}T^{3/4})\\ &\quad + (T^2F + TF^{1/2})/(TF^{1/4}F^{1/2}),\end{aligned} \qquad \text{(C6)}$$

where SD_{AMP} is the standard deviation of movement spatial variability.

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